Parametrii geometrici

ai zonelor cu câmp 2D puternic neuniform:

Colțul si stricțiunea

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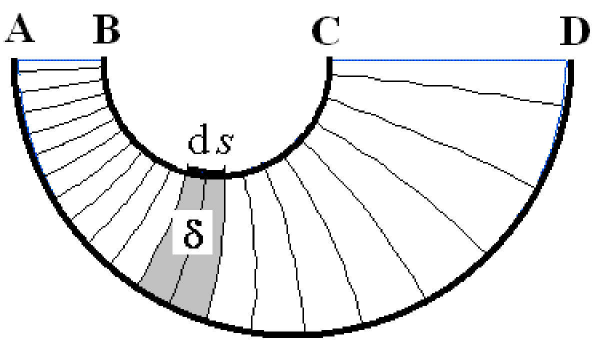
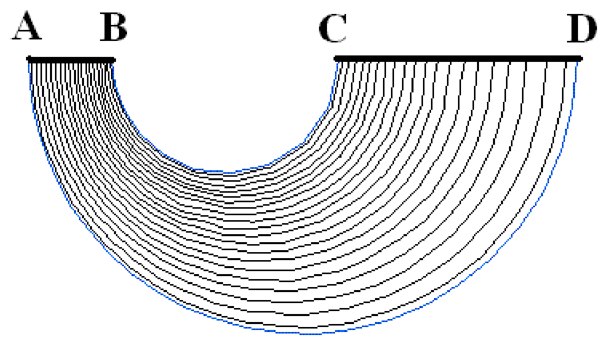
Conductanta geometrica in 2D





$$g_{\mathrm{BC-AD}} = \int_{\mathrm{B}}^{\mathrm{C}} \frac{\mathrm{d}s}{\delta} = r_{\mathrm{AB-CD}}$$

In campuri planparalele si medii omogene si izotrope, conductanta geometrica transversala pe 1 m adancime dintre suprafetele corespondente BC si AD este egala cu rezistenta geometrica longitudinala dintre AB si CD:

Conductanta [S/m]	$G_{\mathrm{BC-AD}} = \sigma\, g_{\mathrm{BC-AD}}$	$R_{\mathrm{AB-CD}} = \rho\, g_{\mathrm{BC-AD}}$	Rezistenta [Ω m]
Permeanta [H/m]	$\Lambda_{\mathrm{BC-AD}} = \mu\, g_{\mathrm{BC-AD}}$	$R_{\mathrm{m\,AB-CD}} = \frac{1}{\mu} g_{\mathrm{BC-AD}}$	Reluctanta [m/H]
Capacitatea [F/m]	$C_{\mathrm{BC-AD}} = \varepsilon\, g_{\mathrm{BC-AD}}$	$R_{\mathrm{T\,AB-CD}} = \frac{1}{\lambda} g_{\mathrm{BC-AD}}$	Rez. termica [m K/W]

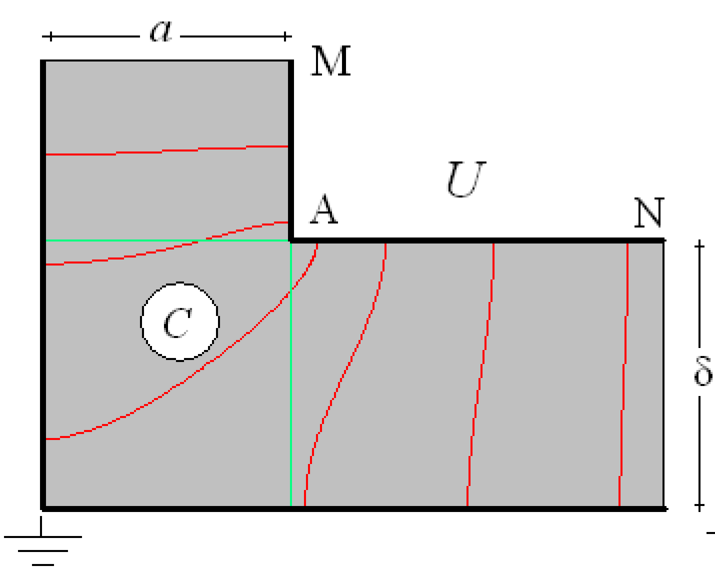
Parametrii (geometrici) ai coltului de 90^0 [81]

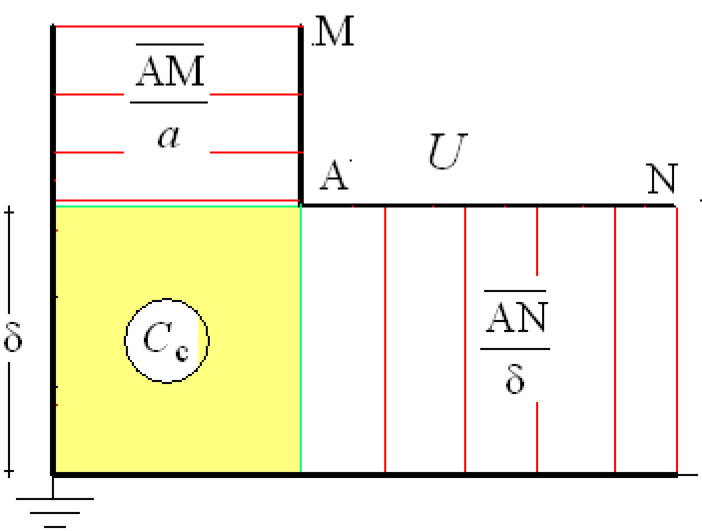
Conductanta, capacitatea sau permeanta transversala (ori rezistemta, reluctanta longitudinala) dintre MAN si masa, pentru $\mathrm{AM}>a$ si $\mathrm{AN}>\delta$:

$$C=\frac{\overline{\mathrm{AM}}}{a}+\frac{\overline{\mathrm{AN}}}{\delta}+C_{\mathrm{c}}$$

$$C_{\mathrm{c}\infty}(x)=\frac{2}{\pi}\left[\frac{\arctan(x)}{x}+x\arctan\left(\frac{1}{x}\right)+\ln\frac{x^2+1}{4x}\right];\quad x=\frac{\delta}{a}$$

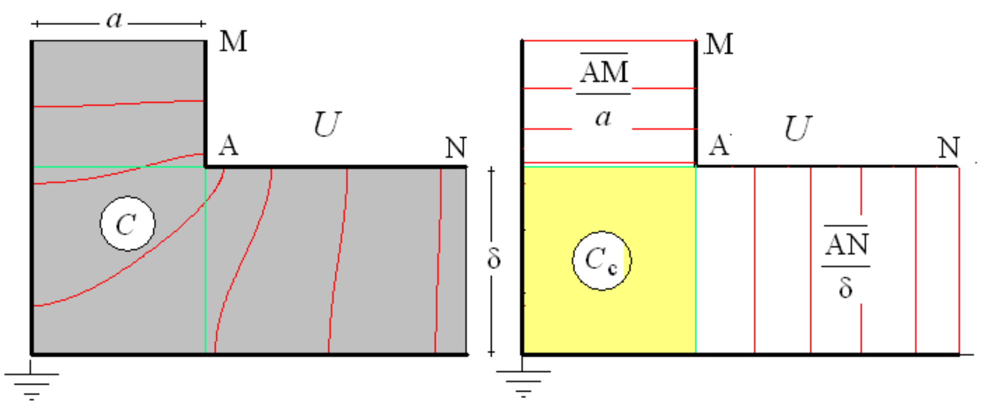
$$\frac{\mathrm{d}C_{\mathrm{c}\infty}}{\mathrm{d}x}=\frac{2}{\pi}\left[\arctan\left(\frac{1}{x}\right)-\frac{\arctan(x)}{x^2}\right]$$





Cazurile AN=∞ si AN=0 [85]

AN >> δ



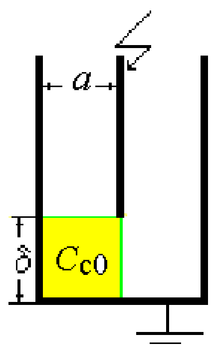
Relatii exacte

$$R_c\big|_{\alpha=90^\circ}(x) = R_c\big|_{\alpha=90^\circ}\left(\frac{1}{x}\right) = C_{c\infty}(x) =$$

$$\frac{2}{\pi}\left[\frac{\arctan(x)}{x} + x\arctan\left(\frac{1}{x}\right) + \ln\frac{x^2+1}{4x}\right]$$

$$\frac{dC_{c\infty}}{dx} = \frac{2}{\pi}\left[\arctan\left(\frac{1}{x}\right) - \frac{\arctan(x)}{x^2}\right]$$

AN = 0

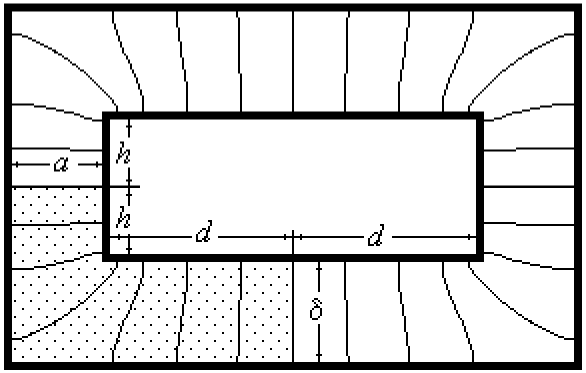


$$C_{c0}(x) = x + \frac{1}{\pi}\ln\frac{2}{\mathrm{ch}(\pi x) - 1}; \quad x = \frac{\delta}{a}$$

$$C'_{c0}(x) = \frac{dC_{c0}}{dx} = 1 - \frac{\mathrm{sh}(\pi x)}{\mathrm{ch}(\pi x) - 1}; \qquad x = \frac{\delta}{a}$$

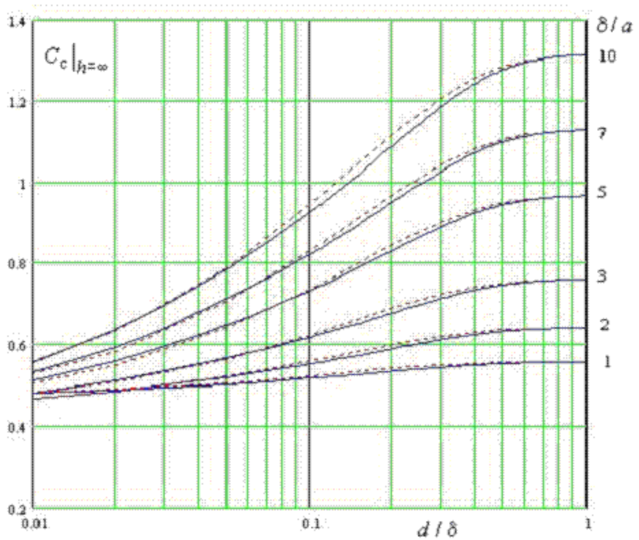
$$\lim_{\delta/a \to \infty} C_{c0} = \frac{2}{\pi}\ln 2 = 0.44$$

Dependenta de *d*/δ <1 si δ/*a* [116]



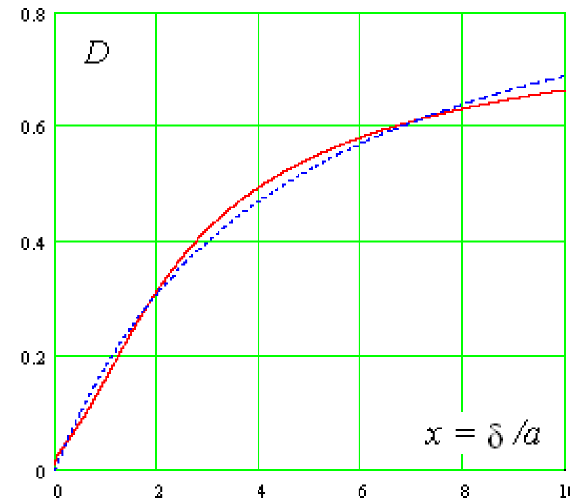
$$C_c\big|_{h=\infty} \approx C_{c\infty}\left[1 - D\cdot\exp\left(-\frac{10d}{\delta\left(0.16 - e^{-10d/\delta}\right)}\right)\right]$$

$$D = 1 - \frac{C_{c0}}{C_{c\infty}} \approx \frac{\delta}{4.5\,a + \delta}; \quad \frac{\delta}{a} < 10$$



dots – FEM

For $d > \delta$ we can consider $d/\delta = \infty$

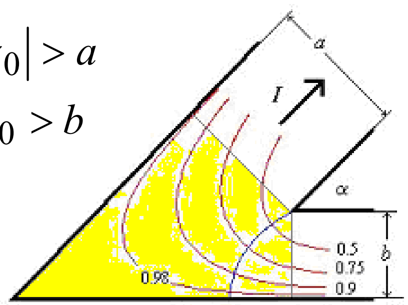


Rezistenta coltului la α arbitrar,

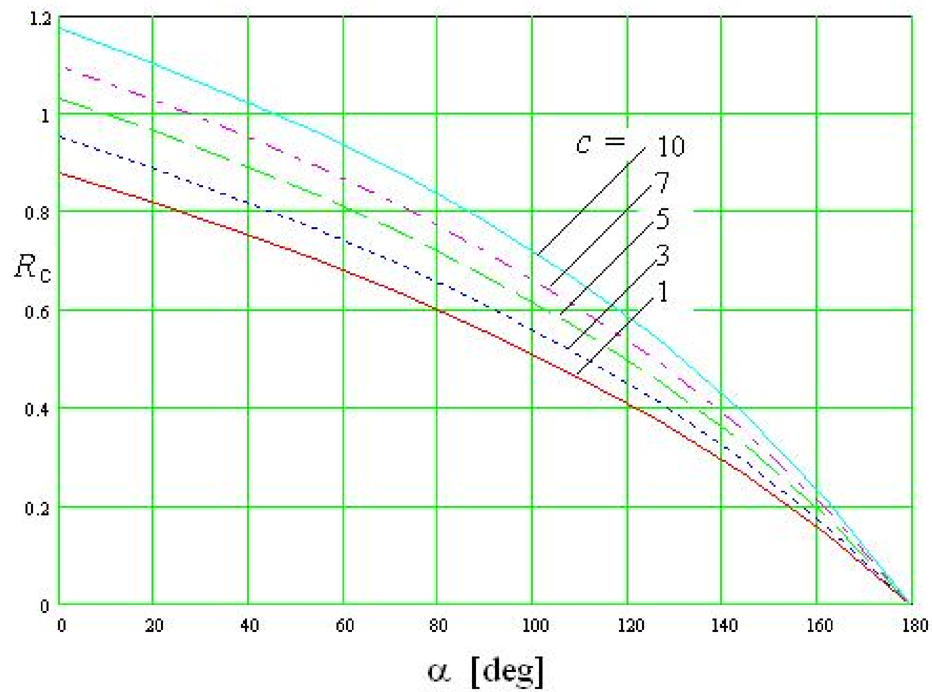
$$c = \left(\frac{a}{b}\right)^{\frac{\pi}{\pi-\alpha}}$$ - parametru [128]

$|z_0| > a$

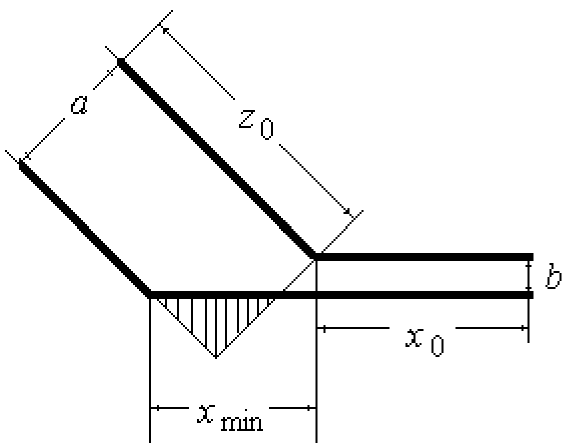
$x_0 > b$



$$\beta = 1 - \alpha/\pi; \quad a = c^{\beta}\, b; \quad c > 1$$



$$R = \frac{1}{\sigma d}\left[\frac{|z_0|}{a} + \frac{x_0}{b} + R_c(\alpha, c)\right] \quad [\Omega]$$



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Rezistenta coltului $R_c(\beta, c)$ [137]

Exact

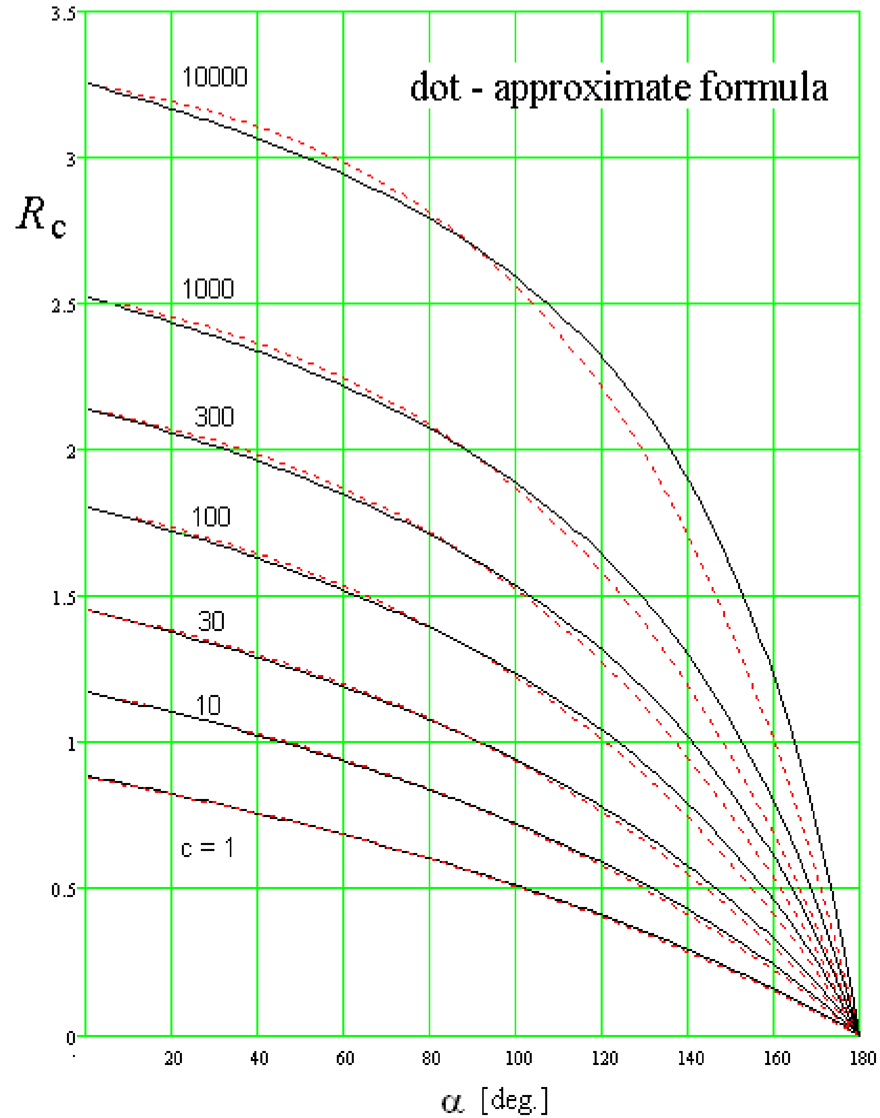
$$I(R,c,\beta) = \int_0^{\ln R} \left(\frac{e^x - 1}{e^x + c}\right) \mathrm{d}x$$

$$R_c(\beta,c) = \frac{1}{\pi}\left[2\cdot\ln(10\cdot c) + \frac{\beta}{10}\left(1+\frac{1}{c}\right)^2 - I(10\cdot c, c, \beta) - I(10\cdot c, 1/c, \beta)\right]$$

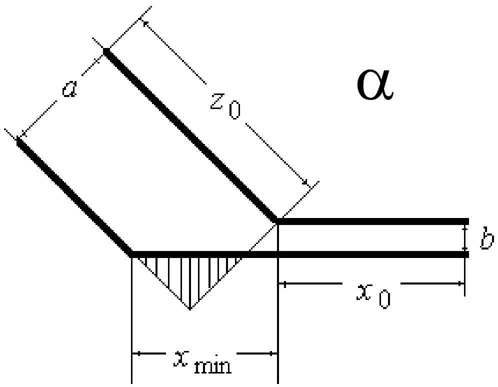
Aproximativ

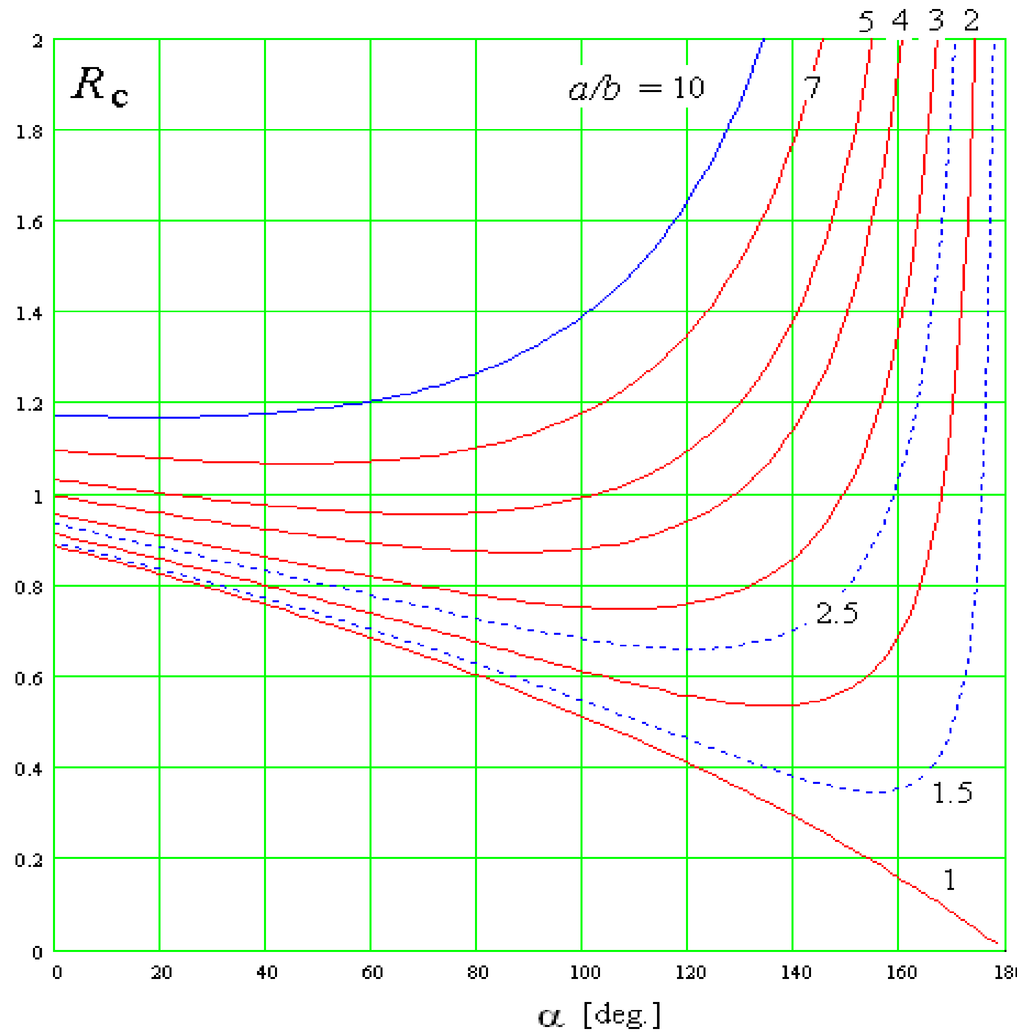
$$c = x^{1/\beta}; \quad z(c) = \frac{R_c\big|_{\alpha=0^\circ}(c)}{R_c\big|_{\alpha=90^\circ}(\sqrt{c})},$$

$$R_c(\beta,c) \approx R_c\big|_{\alpha=0^\circ}(c)\,\frac{1-[z(c)-1]^{2\beta}}{1-[z(c)-1]^2}$$



Rezistenta coltului $R_c(\alpha, x)$ [137]





Lipsa partii hasurate rxplica cresterea lui R_c la $\alpha > 90$ deg si $a > b$

$$R_{c1}(\alpha, x) = R_c\left(1-\frac{\alpha}{\pi}, x^{\frac{\pi}{\pi-\alpha}}\right); \quad x = \frac{a}{b} \geq 1$$

Rezistenta coltului $R_c(\beta, x)$ [137]

$$R_c(\beta,x)=\frac{1}{\pi}\left[3\cdot\ln(10)+\frac{\ln(x)}{\beta}-\int_0^{\ln(10)+\frac{\ln(x)}{\beta}}\left(\frac{e^u-1}{e^u+x^{1/\beta}}\right)^{\beta}\mathrm{d}u-x\int_{0.01}^{1}\left(\frac{1-t}{x^{1/\beta}+t}\right)^{\beta}\frac{\mathrm{d}t}{t}+0.11\cdot\beta\left(1+x^{-1/\beta}\right)\right];\quad x=\frac{a}{b}\geq 1$$

$$\mathrm{Rc}(\beta,\mathrm{x}):=\frac{1}{\pi}\cdot\left[2\cdot\ln\left(10\cdot \mathrm{x}^{\frac{1}{\beta}}\right)-\mathrm{I}\left(10\cdot \mathrm{x}^{\frac{1}{\beta}},\mathrm{x}^{\frac{1}{\beta}},\beta\right)-\mathrm{I}\left(10\cdot \mathrm{x}^{\frac{1}{\beta}},\mathrm{x}^{\frac{-1}{\beta}},\beta\right)+\beta\cdot\frac{1}{10}\cdot\left(1+\mathrm{x}^{\frac{-1}{\beta}}\right)^{2}\right]$$

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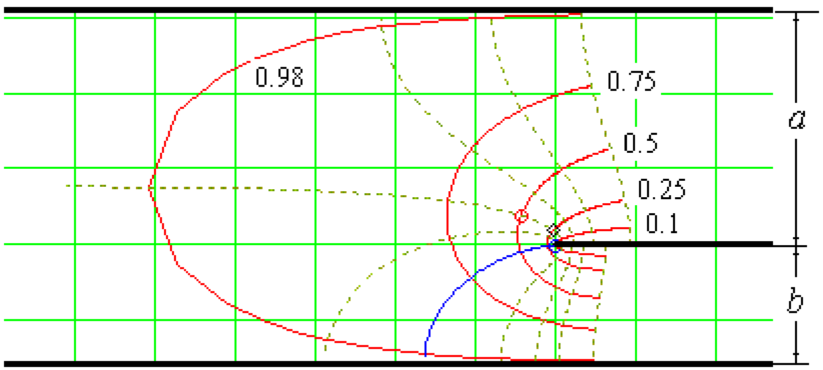
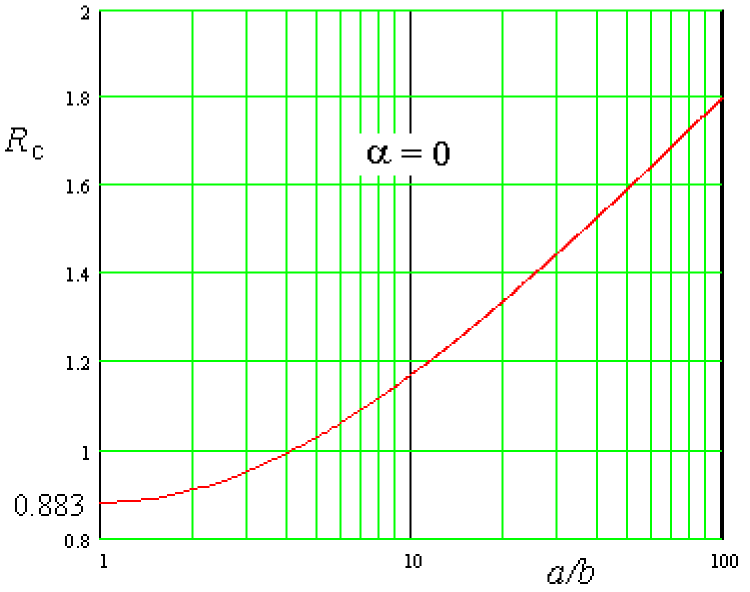
Cazul α = 0° [128]



$$R_c\big|_{\alpha=0} = \frac{x+1}{\pi}\ln\left[\frac{1}{x}(x+1)^{1+1/x}\right]; \qquad x = \frac{a}{b} \geq 1$$



Formule exacte

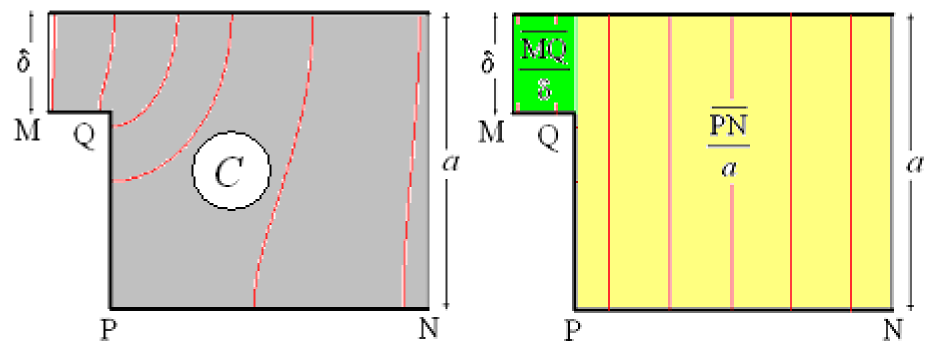
$$R_c(0,c) = \frac{c+1}{\pi}\ln\left[\frac{1}{c}(c+1)^{1+1/c}\right] =$$

$$\frac{a+b}{\pi b}\ln\left[\frac{b}{a}\left(\frac{a}{b}+1\right)^{1+b/a}\right] = R_c(0,1/c)$$

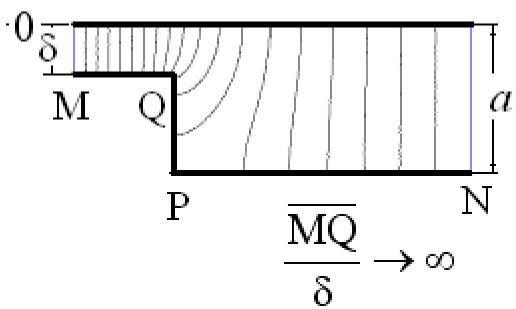
Parametrii “strictiunii” [81]

- Parametrul de “strictiune” este termenul ce trebuie adaugat la valoarea parametrului calculata fara a considera curbura liniilor de camp. El poate fi evaluat printr-o transformare conforma.

$$C = \frac{\overline{PN}}{a} + \frac{\overline{MQ}}{\delta} + C_s$$



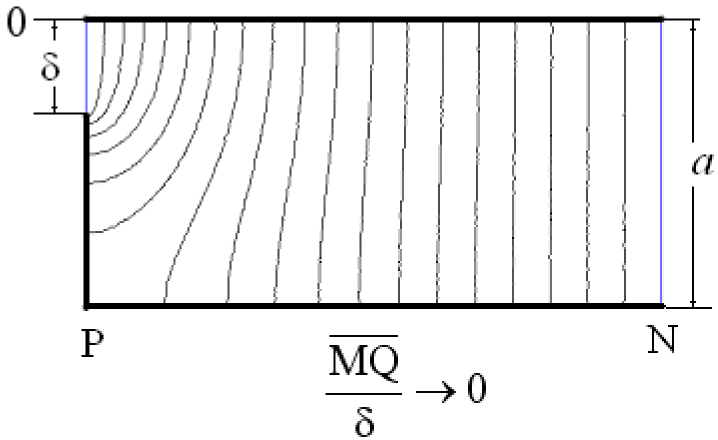
Permeanta de strictiune la PN/*a* = ∞ [85]



$$\lambda_s(x) = \frac{1}{\pi x}\left[(1+x^2)\ln\frac{1+x}{1-x} + 2x\ln\frac{1-x^2}{4x}\right] \qquad \begin{cases} \overline{PN} > a \\ \overline{MQ} > \delta \end{cases}$$

$$\lambda'_s(x) = \frac{d\lambda_s}{dx} = \frac{1}{\pi}\left[1 - \frac{1}{x^2}\right]\ln\frac{1+x}{1-x}; \qquad x = \frac{\delta}{a}$$

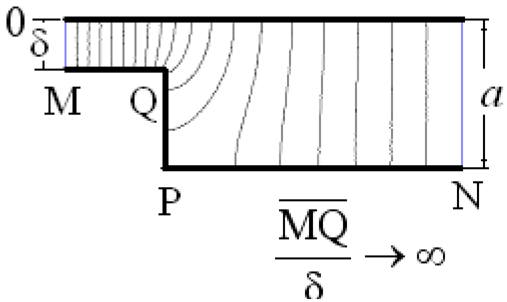
$$\lambda_{MQPN-0} = \frac{\overline{MQ}}{\delta} + \frac{\overline{PN}}{a} + \lambda_s = r_{M0-N0}$$

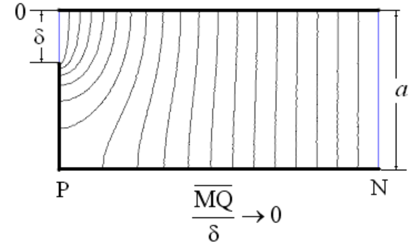


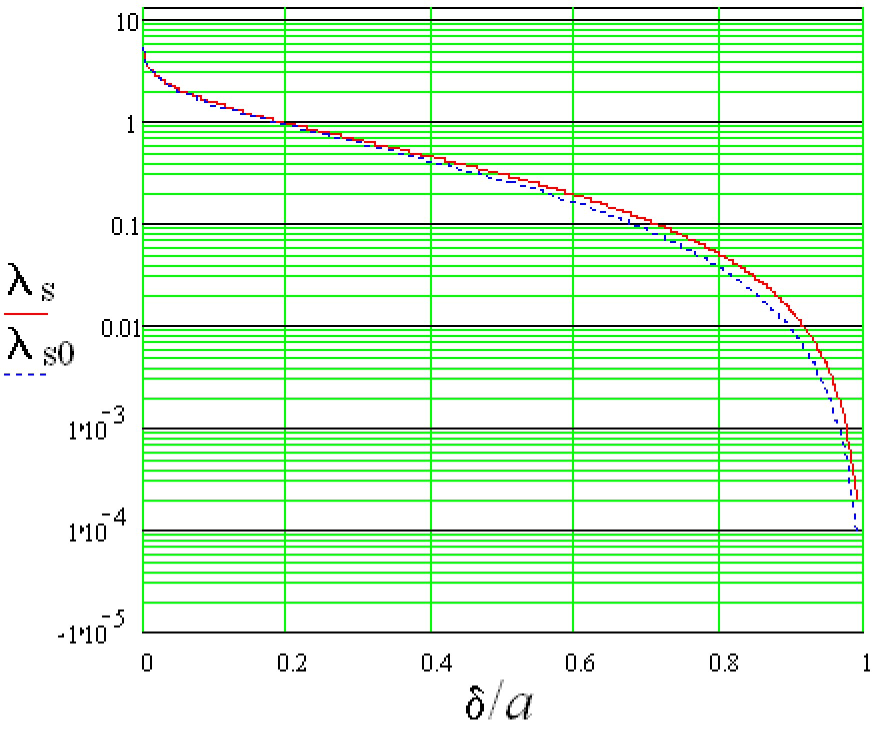
$$\lambda_{s0} = \frac{1}{\pi}\ln\frac{2}{1-\cos(\pi x)} \qquad \begin{cases} \overline{PN} > a \\ \overline{MQ} = 0 \end{cases}$$

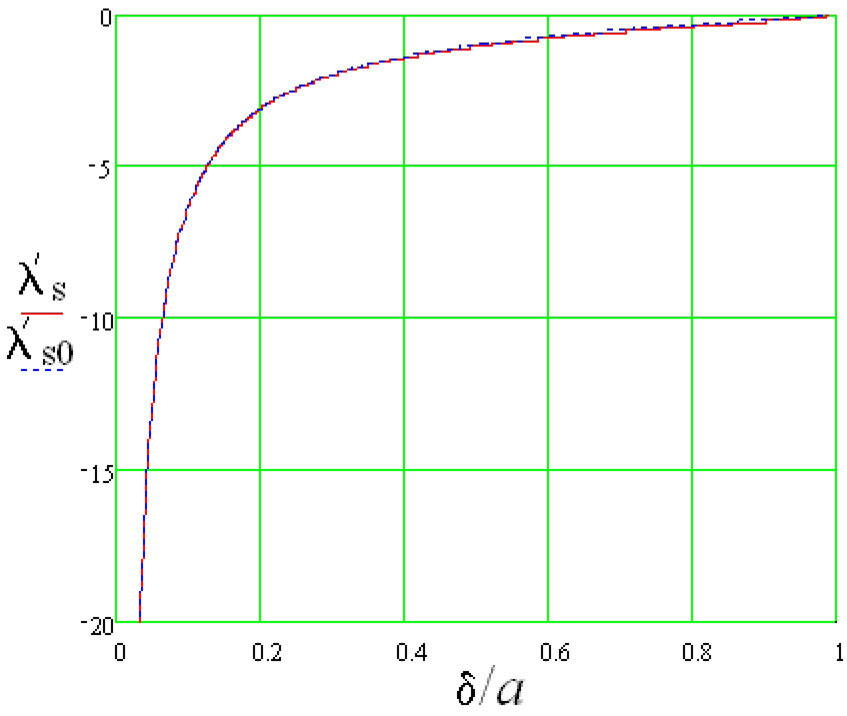
$$\frac{\partial\lambda_{s0}}{\partial x} = -\frac{\sin(\pi x)}{1-\cos(\pi x)}$$

Permeanta de strictiune si derivata ei la MQ/δ = ∞ si 0 [85]

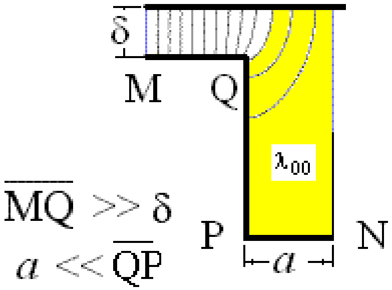


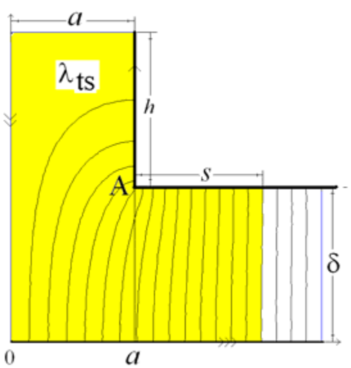


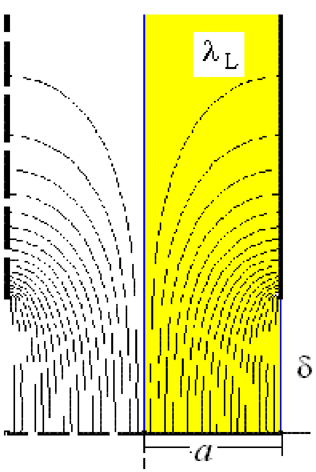




Pemeanta laterală la crestatura adanca PN/QP → 0   [139]







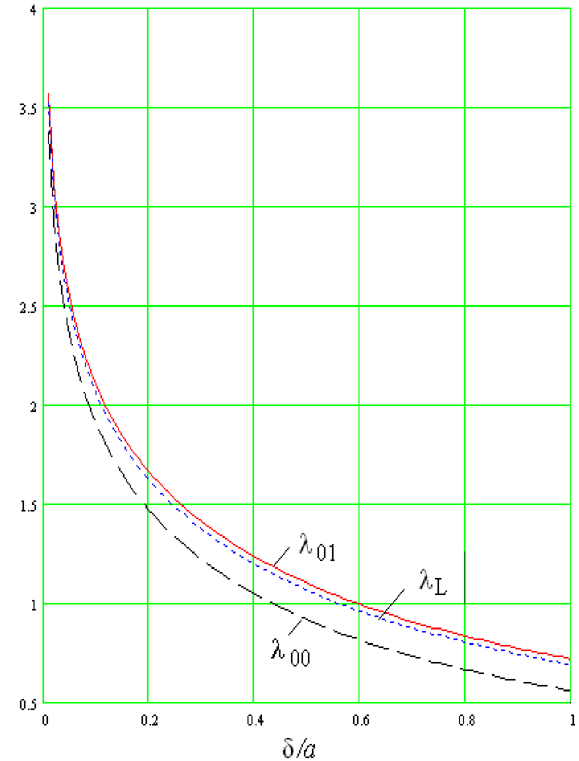
$$\lambda_{00}(x) = \frac{1}{\pi}\operatorname{arccosh}\left(1 + \frac{2}{x^2}\right) \qquad x = \frac{\delta}{a}$$

$$\lambda'_{00}(x) = \frac{d\lambda_{s00}}{dx} = \frac{-2}{\pi x \sqrt{1 + x^2}}$$

$$\lambda_{ts} = \frac{s}{\delta} + \lambda_{01}; \quad s > \delta$$

$$\lambda_{01} = \frac{1}{\pi}\left[\ln\left(1 + \frac{1}{x^2}\right) + \frac{2}{x}\arctan(x)\right]$$

$$\lambda_L = \frac{K(k)}{K(k')}; \quad k' = \tanh(\pi x / 2)$$



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Deep Slot Constriction Permeance [124, 125]

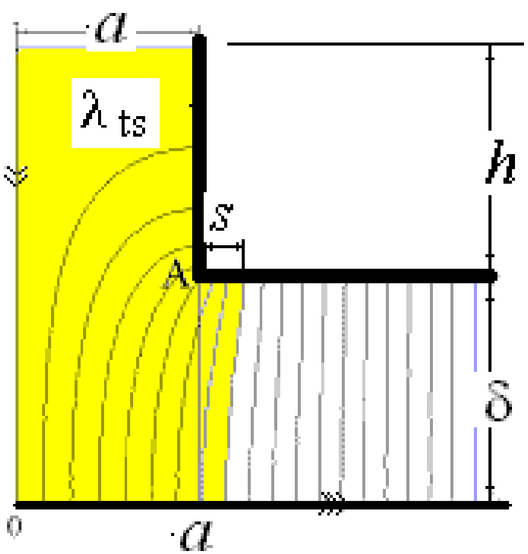
$$h > a$$

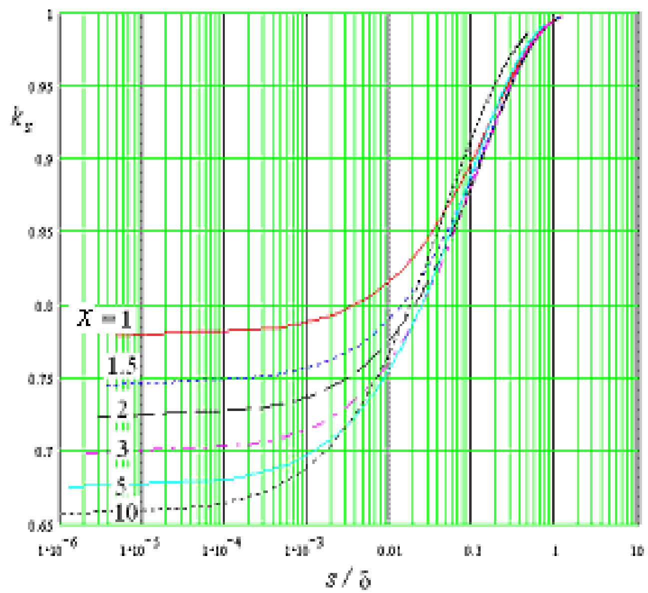
$$\lambda_{ts} = \frac{s}{\delta} + \lambda_{0s}$$

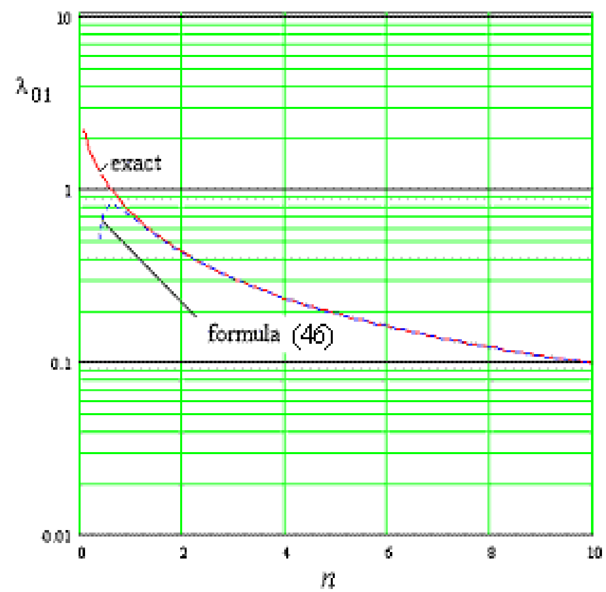
$$\lambda_{01} \cong \frac{1}{n} - \frac{1}{\pi x^2}; \qquad (46)$$

$$\lambda_{0s}(x,s) = k_s(x, s/\delta)\lambda_{01}(x) = \begin{cases} \lambda_{01}(x), & s \to \infty \\ \lambda_{00}(x), & s \to 0 \end{cases}$$

$$\lambda'_{01} \cong \frac{1}{x^2}\left(\frac{2}{\pi x} - 1\right); \qquad x = \frac{\delta}{a} > 1$$







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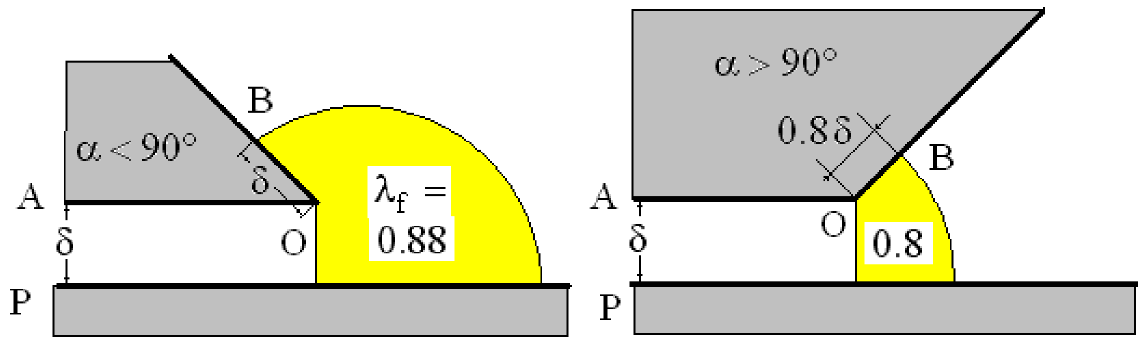
Permeanta de umflari [123]

Permeanta geometrica pana la B

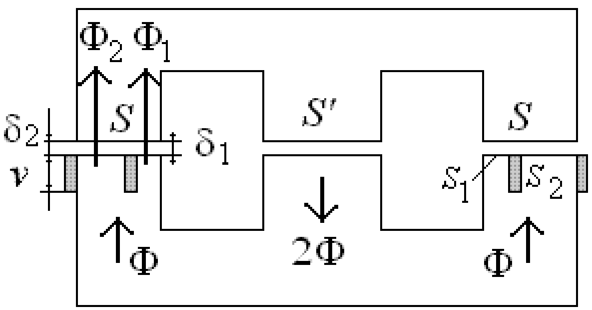


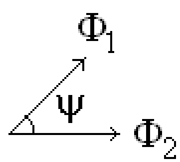
Maximul fortei minime la electromagnetii tip E cu flux constant si spire in scurt circuit [1]

$$F_{\min} = \frac{\Phi^2}{2\mu_0 S} k_F(\alpha, m, \psi)$$

$$\tan\psi_{\max} = \frac{\alpha + m}{\alpha}\sqrt{\frac{1 - \frac{\varepsilon p}{2}(1 - \alpha p)}{1 + \frac{\varepsilon}{2\alpha}(1 - \alpha p)}}$$

$$k_{F\max} = \frac{2(1 + \alpha p)^2}{4\alpha + \varepsilon(1 - \alpha p)^2}; \quad p = \frac{m - 1}{m + \alpha}$$



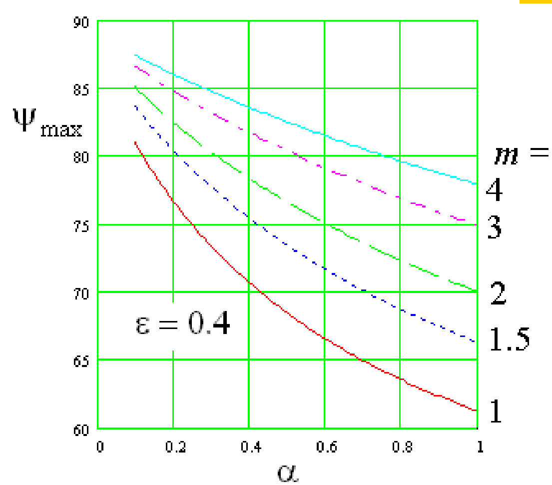


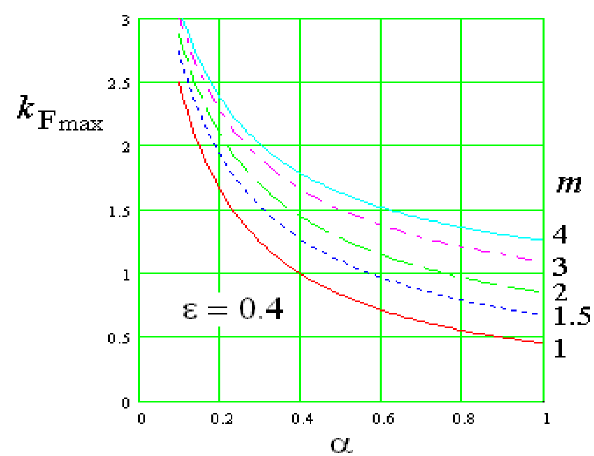
$$\varepsilon = \frac{S'}{S' + 2S}; \quad S = S_1 + S_2$$

$$\alpha = \frac{S_1}{S_2}; \quad \delta'_i = \delta_i + \frac{v}{\mu_i}; \quad i = 1, 2$$

$$m = \frac{\delta'_1}{\delta'_2}; \quad \Phi = \Phi_1 + \Phi_2$$

Elm U transl: $\varepsilon = 1$





Spira in scurt-circuit (considerarea dispersiei) [10]

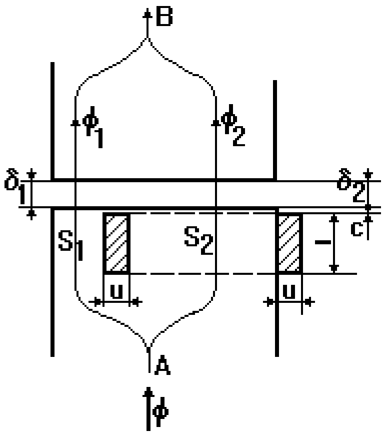


Fig.5.41

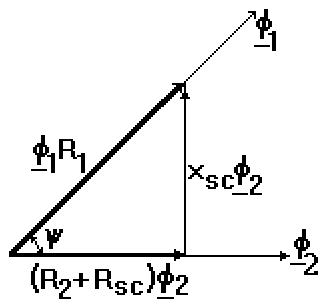


Fig.5.42

$$\delta_i' = \delta_i + \frac{l_i}{\mu_i} \qquad \alpha = \frac{S_1}{S_2}; \quad m = \frac{\delta_1'}{\delta_2'}; \quad \varepsilon = \frac{S'}{S' + 2S}$$

$$\underline{\Phi}_1 R_1 + \underline{\Phi}_2 R_2 = \underline{\hat{I}}_m \qquad R_i = \frac{\delta_i'}{\mu_0 S_i}; \quad i = 1{,}2$$

$$\chi = \frac{x}{\omega \Lambda_2}; \quad \Lambda_2 = \frac{1}{R_2}$$

$$\underline{\hat{I}}_m = \frac{\hat{E}}{r + jx} = \frac{-j\omega\Phi_2}{r + jx} = -\left(\frac{\omega x}{r^2 + x^2} + j\frac{\omega r}{r^2 + x^2}\right)\Phi_2 = -(R_{sc} + jX_{sc})\Phi_2$$

$$\tan\psi = \frac{X_{sc}}{R_2 + R_{sc}}$$

$$\tan\psi = \frac{r}{x}\frac{1}{1 + \chi\left(1 + \frac{r^2}{x^2}\right)} \approx \frac{\omega\Lambda_2}{r}$$

$$\underline{\Phi}_1 R_1 = [(R_2 + R_{sc}) + jX_{sc}]\underline{\Phi}_2$$

$$R_{sc} = \frac{\omega x}{r^2 + x^2}; \qquad X_{sc} = \frac{\omega r}{r^2 + x^2}$$

$$\tan\psi_M = \frac{1}{2\sqrt{\chi(1 + \chi)}}$$

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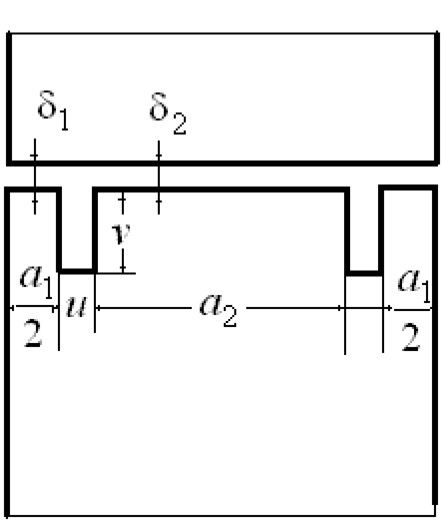
Inductanța de dispersie a spirei in scurt-circuit (in crestătura) [9], [23]



$$\delta'_i = \delta_i + \frac{v_i}{\mu_i} \quad \alpha = \frac{a_1}{a_2} \quad k_1 = \frac{\delta'_1}{\delta'_1 + \alpha\,\delta'_2} \quad ; k_3 = \frac{\delta_1}{\delta'_1 + \alpha\,\delta'_2}\left(1 - \frac{1}{\mu_1}\right)$$

$$h = \alpha\frac{\delta_2}{\delta_1}\frac{1 - 1/\mu_2}{1 - 1/\mu_1} \quad \lambda = \frac{\pi}{v_1 + \delta_1} \quad \varphi = \lambda\,u \quad \theta_i = \lambda\,\delta_i \quad i = 1,2$$

$$S = \sum_{n=1}^{N}\frac{\cos(n\,\theta_1)}{n^2\,\sinh(n\,\varphi)}\left[\left(h\frac{\delta_1}{\delta_2}\sin(n\,\theta_2)\right) + \cosh(n\,\varphi)\sin(n\,\theta_1)\right]$$

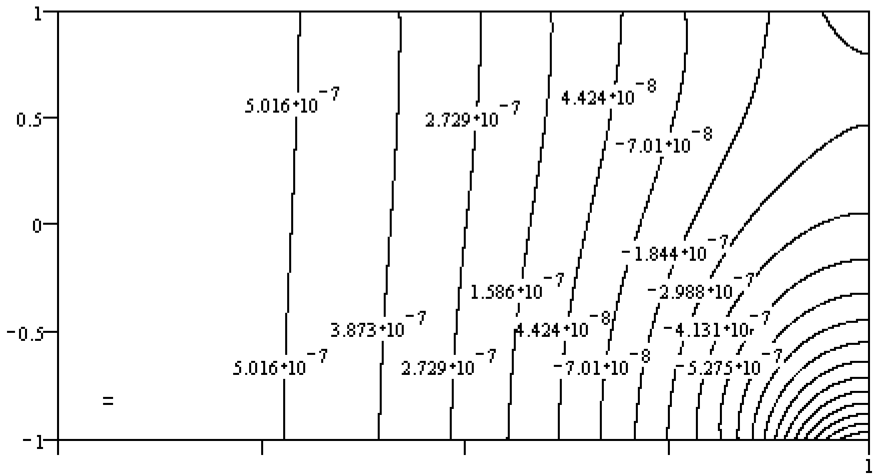
$$L_c = \frac{\mu_0}{2\,\pi}\left[\varphi\left(k_1 - \frac{1}{3}\right) + 4\frac{k_3}{\theta_1}S\right] \quad [\mathrm{H/m}]$$

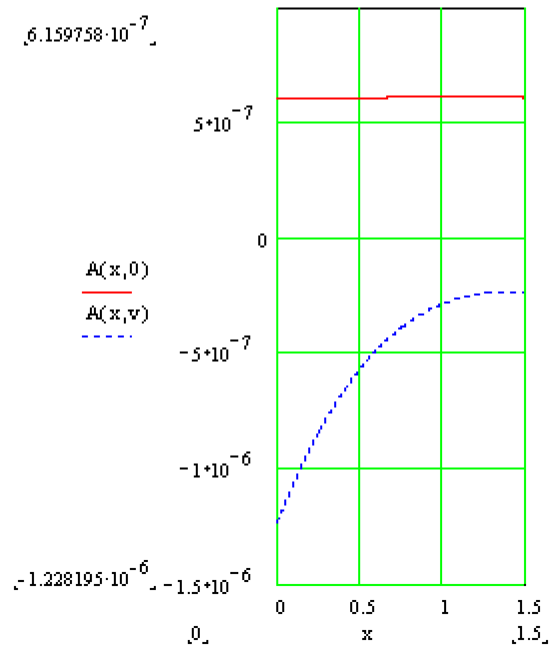
$$N > \frac{1}{2\sqrt{\theta_1\,S\,\varepsilon\%}}$$

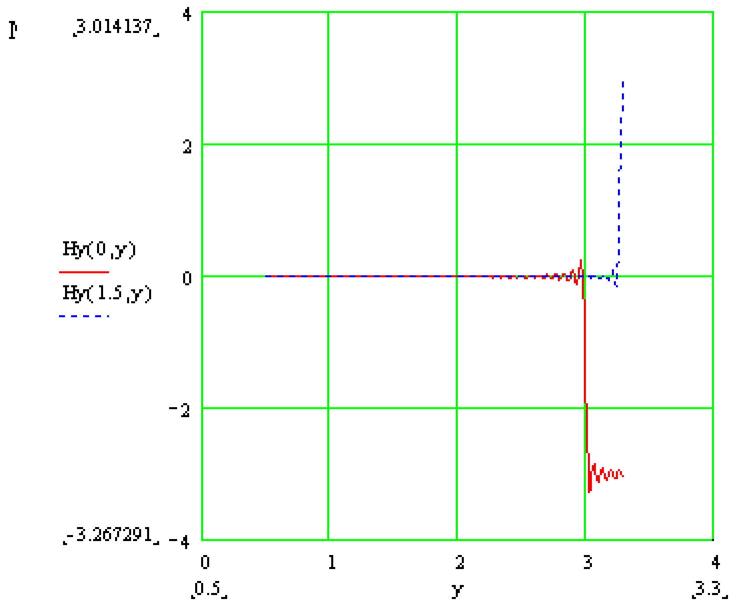
Campul magnetic in crestatura [23]

$$A(x,y)=\mu_0 I\left[\frac{x(2uk_1-x)}{2u(v_1+\delta_1)}-\frac{2k_3}{\pi\theta_1}\sum_{n=1}^{N}(-1)^n\frac{\cos(\lambda n y)}{n^2\sinh(\lambda n u)}\left(h\frac{\delta_1}{\delta_2}\sin(n\theta_2)\cosh(\lambda n x)+\sin(n\theta_1)\cosh(\lambda n(u-x))\right)\right.$$

$$H_y(x,y)=I\left[\frac{x-uk_1}{u(v_1+\delta_1)}+\frac{2k_3}{\pi\theta_1}\sum_{n=1}^{N}(-1)^n\frac{\lambda\cos(\lambda n y)}{n\sinh(\lambda n u)}\left(h\frac{\delta_1}{\delta_2}\sin(n\theta_2)\sinh(\lambda n x)+\sin(n\theta_1)\sinh(\lambda n(u-x))\right)\right]$$







$$a = \begin{bmatrix} 3.6 \\ 4.2 \end{bmatrix} \quad \delta = \begin{bmatrix} 0.3 \\ 0.03 \end{bmatrix} \text{mm} \quad \mu = \begin{bmatrix} 500 \\ 2000 \end{bmatrix}$$

$$u = 1.5 \quad v = 3 \quad a' = 16.4 \quad b = 19.5 \text{ mm}$$

Vibrația si desprinderea armaturii [2]

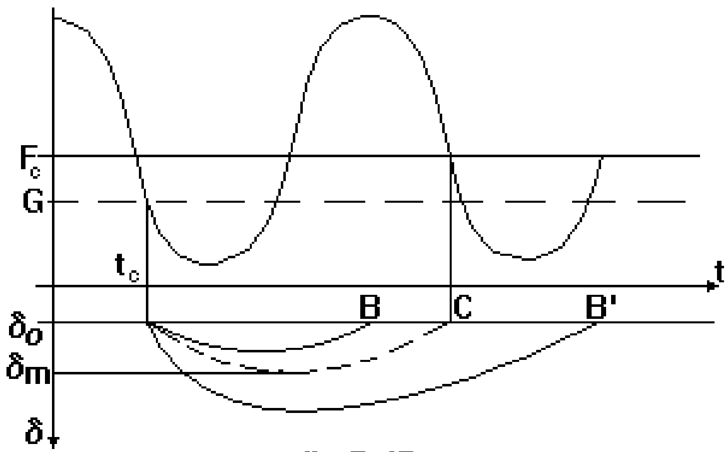


fig.5.47

$$\left|\delta_m - \delta_0\right| \cong 5{,}3 \cdot 10^{-6} \frac{F_\mathrm{v}}{M}$$

$$\frac{0{,}7 F_\mathrm{c}^0}{1 + 0{,}52(\delta_m - \delta_0)\dfrac{a+b}{ab}} \le G_\mathrm{d} \le F_\mathrm{c}(1 - 0{,}3\,n)$$

Condiții tehnice pt. Elm aparatelor de comanda

	c.c.	c.a.
Inchidere	0.85 U_n	0.85 U_n
Deschidere	0.15 U_n	0.35 U_n

$$G \le F_{\mathrm{min}}$$