ICATE 2002

UNIVERSITY OF CRAIOVA
FACULTY FOR ELECTRICAL ENGINEERING
FACULTY FOR ELECTROMECHANICAL ENGINEERING

PROCEEDINGS

ON THE

INTERNATIONAL CONFERENCE ON APPLIED AND THEORETICAL ELECTRICITY

CRAIOVA / BAILE HERCULANE, OCTOBER 17 – 19, 2002

ROMANIA

Section 3 ELECTRICAL DRIVES

Section 4 ELECTRICAL APPARATUS. HIGH VOLTAGE EQUIPMENTS

INDUCTANCE OF A PLUNGER-TYPE ELECTROMAGNET

Grigore A. CIVIDJIAN, Alin DOLAN, Victor CLIMOV, Ali Ihsan CANAKOGLU[*]

University of Craiova, Romania,[*] Dumlupinar University, Kutahya, Turkey
Lapus 5, RO-1100 Craiova, Tel: +40251/435724, E-mail : gcividjian@elth.ucv.ro

Abstract
In the paper, an experimental and FEM verification of early obtained, internal inductance formula for plunger type magnet is performed. A new, faster, method for the calculation using Mathcad is substantiate and recommended.

1. - INTRODUCTION

In [1] a formula for internal inductance of plunger-type magnet is deduced, using the method of power approximation of boundary conditions in 2D connected regions. The proposed formula, which takes into consideration the core saturation, the parasite air gaps and the variation in large limits of the working gap, is difficult to apply because of the function

$$F(\alpha, z) = \int_0^1 \frac{\cos[z(1-t)]}{t^\alpha} dt; \quad \alpha \in (0,1) \tag{1}$$

containing an improper integral.

In the paper, a method for the evaluation of this function, using Mathcad, is proposed and two verifications of the formula are performed: using a finite element method and experimental determinations of the total inductance for large range of the gap variation. The experimental values are compared with the sum of internal inductance, calculated with formula given in [1] and external inductance calculated with finite element method (Quickfield and femm).
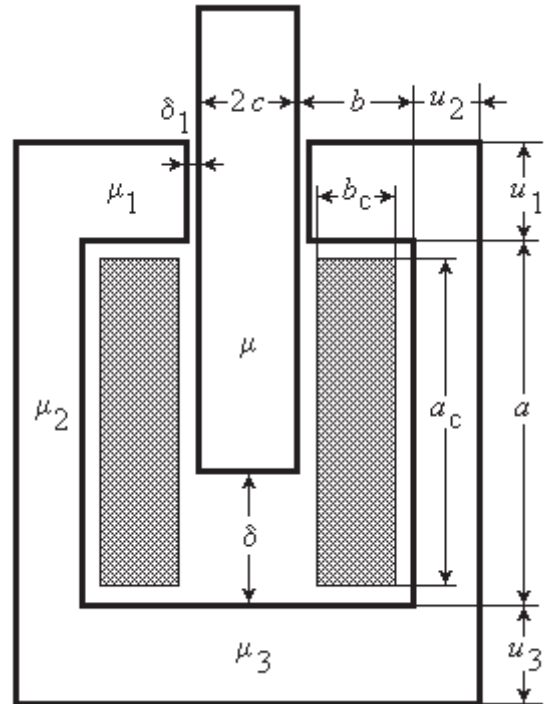
Fig. 1 Plunger - Type Magnet

2.-CALCULATION OF $F(\alpha, z)$

The values of α was taken using the recommendations given in [1]. The recommendations from [2] can be also used.

The function (1) contain an improper integral, difficult to calculate, but it can be expressed in terms of Bohmer integrals (or generalized Fresnel integrals) [3]:

$$C(x,\alpha) = \int_x^\infty \frac{\cos t}{t^{1-\alpha}} dt; \qquad S(x,\alpha) = \int_x^\infty \frac{\sin t}{t^{1-\alpha}} dt$$

$$C(x,\alpha) = \frac{1}{2}\left[e^{-\frac{i\pi\alpha}{2}} \Gamma(\alpha, ix) + e^{\frac{i\pi\alpha}{2}} \Gamma(\alpha, -ix) \right] \tag{2}$$

$$S(x,\alpha) = \frac{1}{2i}\left[e^{\frac{i\pi\alpha}{2}} \Gamma(\alpha, -ix) - e^{-\frac{i\pi\alpha}{2}} \Gamma(\alpha, ix) \right]$$

For $x \to 0$ we have evidently

$$C(0,\alpha) = \Gamma(\alpha)\cos\left(\frac{\alpha\pi}{2}\right);$$
$$S(0,\alpha) = \Gamma(\alpha)\sin\left(\frac{\alpha\pi}{2}\right) \tag{3}$$

We will consider also the complementary integrals

$$C_0(x,\alpha) = \int_0^x \frac{\cos t}{t^{1-\alpha}}dt; \qquad S_0(x,\alpha) = \int_0^x \frac{\sin t}{t^{1-\alpha}}dt \tag{4}$$

They satisfy the equations:

$$C_0(x,\alpha) + C(x,\alpha) = C(0,\alpha) = \Gamma(\alpha)\cos\left(\frac{\alpha\pi}{2}\right)$$
$$S_0(x,\alpha) + S(x,\alpha) = S(0,\alpha) = \Gamma(\alpha)\sin\left(\frac{\alpha\pi}{2}\right) \tag{5}$$

For large x, the following equations can be useful [3]:

$$C(x,\alpha) = -x^{\alpha}\left[P(x,\alpha)\sin x + Q(x,\alpha)\cos x\right]$$
$$S(x,\alpha) = x^{\alpha}\left[P(x,\alpha)\cos x - Q(x,\alpha)\sin x\right] \tag{6}$$

where

$$P(x,\alpha) = \sum_{m=0}^{M-1} \frac{(-1)^m (1-\alpha)_{2m}}{x^{2m+1}} + O\left(|x|^{-2M-1}\right)$$
$$Q(x,\alpha) = \sum_{m=1}^{M} \frac{(-1)^m (1-\alpha)_{2m-1}}{x^{2m}} + O\left(|x|^{-2M-2}\right) \tag{7}$$

Here is used the Pochhammer symbol

$$(\alpha)_n = \alpha(\alpha+1)(\alpha+2)..(\alpha+n-1) = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)} \tag{8}$$

Using the substitution $zt = x \implies z\,dt = dx$ to change the integration variable t with x in (1), the function F can be expressed in terms of the complementary integrals (4)

$$F(\alpha,z) = \frac{1}{z^{1-\alpha}}\int_0^z \frac{\cos(z-x)}{x^{\alpha}}dx =$$
$$= \frac{1}{z^{1-\alpha}}\left[\cos z\, C_0(z,1-\alpha) + \sin z\, S_0(z,1-\alpha)\right] \tag{9}$$

The complementary integrals have the following series representation [3]:

$$C_0(x,\alpha) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m+\alpha}}{(2m)!(2m+\alpha)}$$
$$S_0(x,\alpha) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m+1+\alpha}}{(2m+1)!(2m+1+\alpha)} \tag{10}$$

The above series are alternating and the truncation error is less than the first neglected term. For $x \le 1$, the error, for $(m+1)$ taken terms, is less than

$$T(m) = \frac{1}{2m(2m)!} \tag{11}$$

For error less than 10^{-12} there are enough 7 terms. To avoid the calculation of improper integrals we propose to calculate the complementary integrals (4), using Mathcad, as follows:

$$C_0(x,\alpha) = \begin{cases} C_{01}(x,\alpha) & x \le 1 \\ C_{01}(1,\alpha) + \int_1^x \frac{\cos t}{t^{1-\alpha}}dt & x > 1 \end{cases}$$
$$S_0(x,\alpha) = \begin{cases} S_{01}(x,\alpha) & x \le 1 \\ S_{01}(1,\alpha) + \int_1^x \frac{\sin t}{t^{1-\alpha}}dt & x > 1 \end{cases} \tag{12}$$

where

$$C_{01}(x,\alpha) = \sum_{m=0}^{6} \frac{(-1)^m x^{2m+\alpha}}{(2m)!(2m+\alpha)}$$
$$S_{01}(x,\alpha) = \sum_{m=0}^{6} \frac{(-1)^m x^{2m+1+\alpha}}{(2m+1)!(2m+1+\alpha)} \tag{13}$$

The integrals in equations (12) are not improper and can be easy calculated with good precision by Mathcad. The values of the function F calculated with (9), (12) and (13) are given in fig. 2.
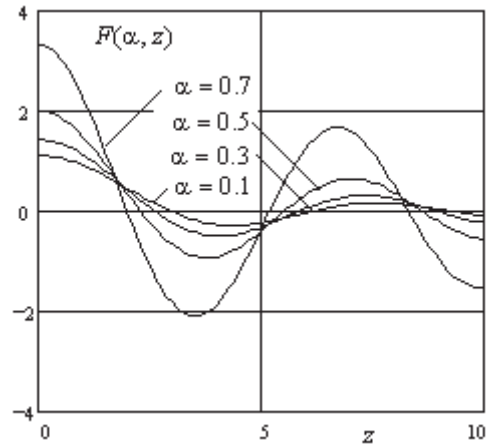
Fig. 2. The function $F(\alpha, z)$

The function F can be easy calculated also using the equations (6). Replacing (6) in (5) and (9) we obtain:

$$F(\alpha,z) = \frac{\Gamma(\alpha)}{z^{1-\alpha}}\cos\left(\frac{\alpha\pi}{2} - z\right) + Q(z,1-\alpha) \tag{14}$$

The last term, in according with (7) can be written as

194

$$Q(z, 1-\alpha) \cong \sum_{m=1}^{M} \frac{(-1)^m \, \Gamma(\alpha + 2m - 1)}{z^{2m} \, \Gamma(\alpha)} \qquad (15)$$

For $z > 10$ it can be calculated with an absolute error less than 10^{-12}, if $M = 5$.

The values of $-Q(z, 1-\alpha)$ are given in fig. 3. We can see that for large z this term can be neglected. In the program given in appendix of [1] this term is neglected, for z larger than 90. This corresponds to an absolute error smaller than 10^{-4}.
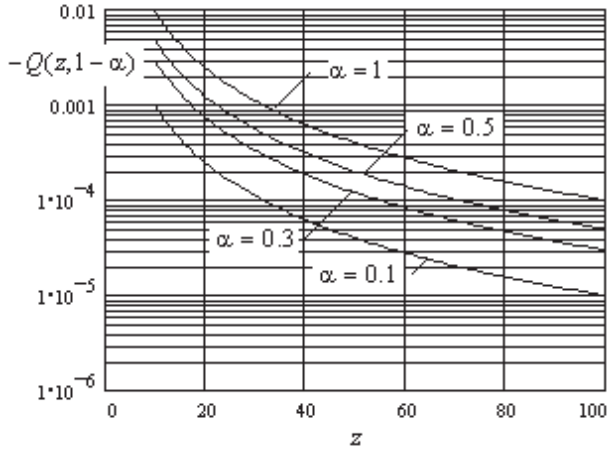

Fig. 3. The function (15)

3. – INTERNAL INDUCTANCE

The inductance corresponding to the magnetic flux in the two windows of the magnet (fig. 1) according to [1], can be written in the form

$$L_{\text{int}} =$$

$$2\mu_0 \, w^2 \left[G + \frac{c}{\delta_e} + \frac{a_1 \, a + b_1 \, b}{2\,\delta_e} - \frac{a_2 \, a^2 + b_2 \, b^2}{6\,a\,b} \right] \qquad (16)$$

In this formula the coil cross-section is considered to have the window dimensions: $a_c \approx a$, $b_c \approx b$.
The equivalent air gap, using the notation from fig. 1

$$\delta_e =$$

$$\delta + \frac{a-b}{\mu} + \frac{c}{u_1}\left(\delta_1 + \frac{b-\delta_1}{\mu_1}\right) + \frac{c}{u_2}\frac{a}{\mu_2} + \frac{c}{u_3}\frac{b}{\mu_3} \qquad (17)$$

The magnetic field in varies portions of the magnet can be approximated neglecting the leakage flux:

$$H_\delta = \mu H = \frac{u_1}{c} H_{\delta_1} = \frac{\mu_i \, u_i}{c} H_i = \frac{w \, I}{\delta_e}; \quad i = 1...3 \qquad (18)$$

The coefficients a_i and b_i have the expressions [1]:

$$a_1 = \frac{c}{\mu_3 u_3}; \quad a_2 = \frac{1}{\delta_e}\left[\frac{c}{u_1}\left(\delta_1 + \frac{b-\delta_1}{\mu_1}\right) + \frac{b}{\mu_3}\frac{c}{u_3}\right] \qquad (19)$$

$$b_1 = \frac{1}{a}\left(\delta + \frac{a-\delta}{\mu}\right); \quad b_2 = \frac{1}{\delta_e}\left[\delta + \frac{a-\delta}{\mu} + \frac{a}{\mu_2}\frac{c}{u_2}\right]$$

The interpolation coefficients can be selected following the recommendations [1]:

$$k_1 = 1 - \frac{1}{6\alpha}; \quad k_2 = \frac{1-\alpha}{6\alpha}$$

$$\alpha = 0.44 + 2.2(\delta/a) - 2.68(\delta/a)^2 \text{ for } b/a = 0.10$$
$$\alpha = 0.42 + 1.80(\delta/a) - 2.20(\delta/a)^2 \text{ for } b/a = 0.15 \qquad (20)$$
$$\alpha = 0.41 + 1.29(\delta/a) - 1.60(\delta/a)^2 \text{ for } b/a = 0.25$$
$$\alpha = 0.44 + 0.70(\delta/a) - 1.15(\delta/a)^2 \text{ for } b/a \geq 0.50.$$

or, for larger gaps, the recommendations [2]:

$$k_1 = 1 - \frac{1-u_0}{\alpha}; \quad k_2 = (1-u_0)\frac{1-\alpha}{\alpha}$$

$$u_0 = 0.831957 + 1.54394 \cdot 10^{-2}\frac{\delta}{b} - 0.175235\left(\frac{\delta}{b}\right)^2 +$$

$$+ 7.66579 \cdot 10^{-2}\left(\frac{\delta}{b}\right)^3 - 1.60908 \cdot 10^{-2}\left(\frac{\delta}{b}\right)^4 +$$

$$+ 1.83817 \cdot 10^{-3}\left(\frac{\delta}{b}\right)^5 - 1.09529 \cdot 10^{-4}\left(\frac{\delta}{b}\right)^6 + \qquad (21)$$

$$+ 2.66537 \cdot 10^{-6}\left(\frac{\delta}{b}\right)^7$$

$$\alpha = 0.3 + 0.021 \cdot \frac{\delta}{b}$$

The proper values result from the equations

$$\lambda_1 a = \lambda_2 b = \nu \, \delta = k \, \pi \qquad (22)$$

The Fourier coefficients are

$$c_k =$$

$$\frac{-2}{k\pi \, \text{th}(\lambda_1 b)}\left[\left(k_1 - \frac{1}{\mu}\right)\frac{\sin(\lambda_1 \delta)}{\lambda_1 \delta_e} + k_2 \frac{\delta}{\delta_e}F(\alpha, \lambda_1 \delta)\right]$$

$$d_k = \frac{-2}{k\pi \, \text{th}(\lambda_2 a)}\frac{c}{u_1}\left[\left(k_1 - \frac{1}{\mu_1}\right)\frac{\sin(\lambda_2 \delta_1)}{\lambda_2 \delta_e} + k_2 \frac{\delta_1}{\delta_e}F(\alpha, \lambda_2 \delta_1)\right] \qquad (23)$$

The constant G from the equation (16) is given in fig. 3 from [1] and can be calculated, taking 10 terms in the equation:

$$G = \frac{a_2 \delta^2}{6ab} - \frac{a_1 \delta}{2\delta_e} -$$
$$- \sum_{k=1}^{\infty} \left[c_k \frac{\sin(\lambda_1 \delta)}{\lambda_1 \delta} + d_k \frac{\mathrm{sh}(\lambda_2 \delta)}{\lambda_2 \delta \, \mathrm{ch}(\lambda_2 a)} \right] \qquad (24)$$

4. FEM AND EXPERIMENTAL VERIFICATION

For FEM and experimental verification the electromagnet shown in fig. 1 and 4, with the dimensions (in mm) given in the next table, was used

a	b	c	u_1	u_2	u_3
70	17.75	7.25	15	9	15

g	δ_1	w	a_c	b_c	R
15.3	0.15	2350	65	13.8	100

where g is the core thickness, a_c, b_c are the coil dimensions, w the number of turns and R the domain radius for external inductance calculation. The relative permeability was considered 1000 for all the portions of the core.

In the next table, are given the 2D magnetic field energy (per 1 m length) W_{ext} in two lateral sides of the coil up to a distance $R = 100$ mm and the magnetic energy W_{int} in the two magnet windows, both calculated by Quickfield program for a apparent current density of 1 A/mm^2, i.e. a coil current

$$I_1 = 10^6 \frac{a_c \, b_c}{w} = 0.382 \, [\mathrm{A}] \qquad (25)$$

In next two columns the corresponding external and internal inductances in H/m are given, calculated with the formulas

$$L'_{ext} = 2 \frac{2 W_{ext}}{I_1^2}; \qquad L'_{int} = 2 \frac{2 W_{int}}{I_1^2} \qquad (26)$$

and in the last column the theoretical values calculated with the formula (16).

	FEM (QFLD, FEMM)				(16)
δ	W_{ext}	W_{int}	L'_{ext}	L'_{int}	L_{int}
[mm]	[J/m]	[J/m]	[H/m]	[H/m]	[H/m]
2.5	0.183	2.355	5.018	64.666	66.211
5	0.173	1.564	4.761	42.941	43.951
10	0.153	1.019	4.209	27.987	28.856
20	0.126	0.654	3.459	17.955	17.228
40	0.066	0.251	1.819	6.882	7.194

We can observe in fig. 7 a good agreement with the FEM data (differences less than 4.5%).

Using the external inductances determined with FEM and the determined by FEM and calculated with equation (16) internal inductance, the total coil inductance was calculated, considering for internal inductance the core thickness g and for external inductance the rest of mean turn half length:

$$L'_{tot} = g L'_{int} + \left(\frac{l_m}{2} - g \right) L'_{ext}$$
$$L_{tot} = g L_{int} + \left(\frac{l_m}{2} - g \right) L'_{ext} \qquad (27)$$
$$l_m = 2(c_c + g_c) + \pi b_c$$

In the last table, these values are compared with experimental data obtained for feeding voltage $U = 50$ V, 50 Hz and the currents I given in the table, considering the coil resistance $R_c = 23.3\ \Omega$:

$$L''_{tot} = \frac{1}{100\pi} \sqrt{\left(\frac{U}{I} \right)^2 - R_c^2} \qquad (28)$$

The larger difference between the experimental data and calculated inductances for small air gap can be explained by the altered shape of the plunger which is not flat faced, lake in fig. 1 and has also a T shape instead of I. The smaller plunger area gives a smaller inductance. This explanation is supported by the good agreement of the formula (16) results with the FEM data.

δ	I	L''_{tot}	L'_{tot}	L_{tot}	L_{tot} / L''_{tot}
[mm]	[A]	[H]	[H]	[H]	
2.5	0.158	1.005	1.214	1.238	1.232
5	0.193	0.821	0.870	0.885	1.078
10	0.251	0.630	0.617	0.630	1
20	0.377	0.416	0.403	0.418	1.007
40	0.814	0.181	0.187	0.191	1.057

For gaps larger than 5 mm the differences are smaller than 8 %, which can be considered acceptable.

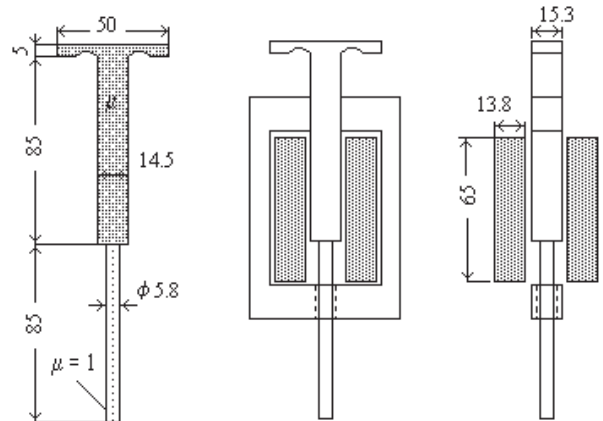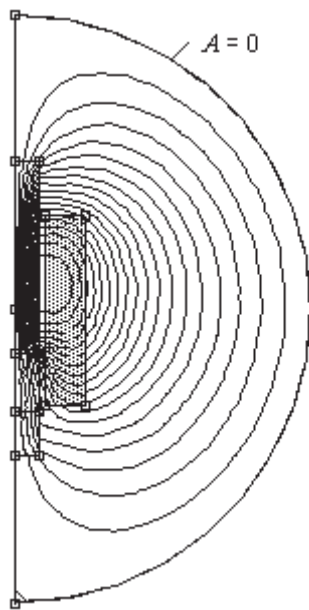
Fig. 4 The experimental model

Fig. 5. External magnetic field for the gap 20 mm

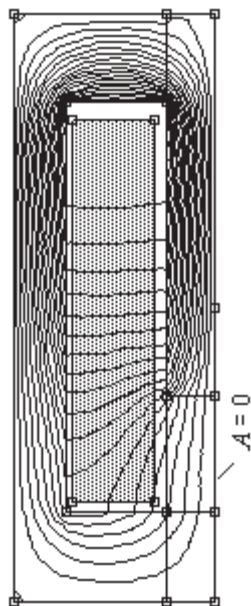$W_{\text{ext}} = 0.126$ J/m, $w\,I = 898$ A

1352 nodes, 2547 elements

Fig. 6 Internal magnetic field for the gap 20 mm

$w\,I = 898$ A, $W_{\text{int}} = 0.654$ J/m
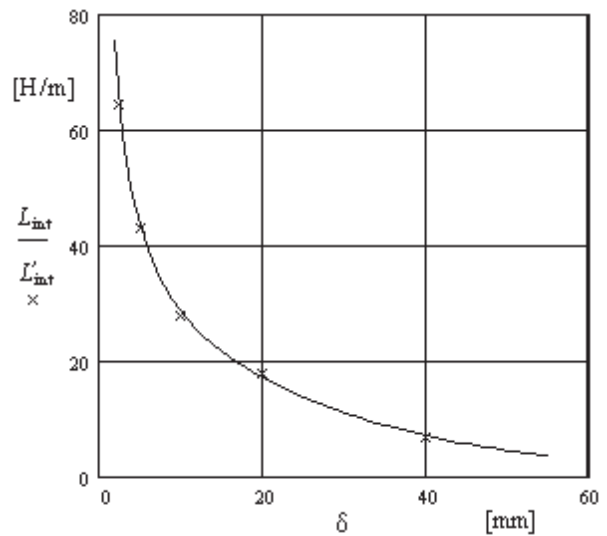
1444 nodes, 2730 elements

Fig. 7. Internal inductance calculated with (16) (line) and determined by FEM (x's)

5. – DELPHI PROGRAM

A user-friendly Delphi program (called IPTM.exe) was built by the last co-author, which determine very fast the constant G and internal inductance L_{int} using the formulas (16) and (24) and given in [1] algorithm for F function calculation.

The program calculates also the vector magnetic potential for given points in the two domains: I – coil cross-section and II – the air gap. The results can be stored in text files and the graphics in bmp. Files. Four (color) menu pages offered by this program are shown in the next several figures.

Fig. 8. Delphi program main menu

197

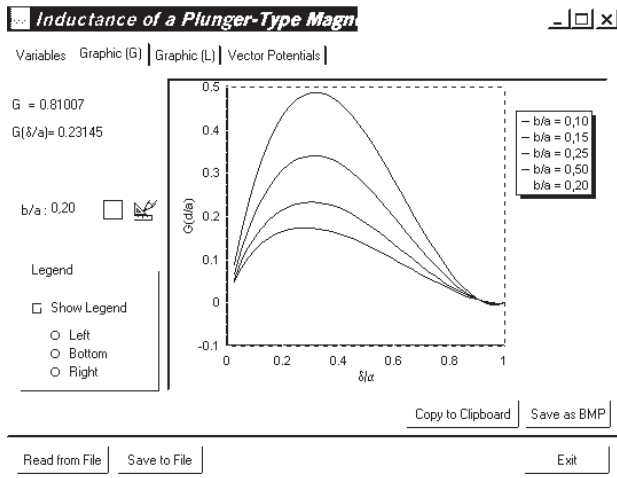In fig. 8 the dimensions of the experimental model are introduced.

Fig. 9. The parameter $G\dfrac{\delta}{a}$ as a function of $\dfrac{\delta}{a}$

for four ratio of window dimensions

In the figure above the constant G is calculated for four shapes of the rectangular window.

The time need to calculate the graphics for G is around 4 seconds, for a computer with 800 MHz processor. The next menus are almost instantaneous.
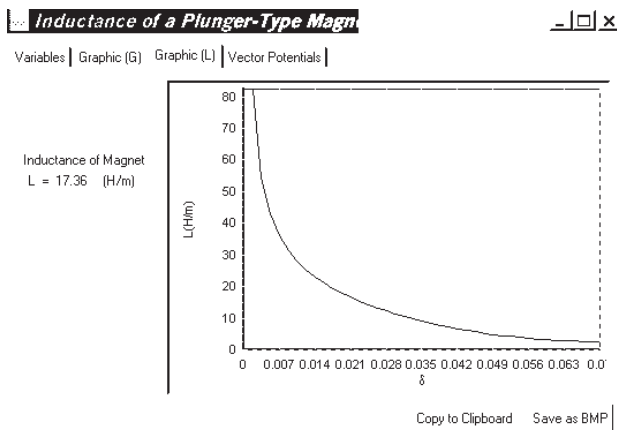

Fig. 10. The internal inductance L_{int} as a function of the gap

The next menu calculates the vector magnetic potential in the two domains (core window and air gap), i. e. the magnetic flux per 1 m depth, between the zero flux line (A = 0 in fig. 6) and the selected point, in [Wb/m].
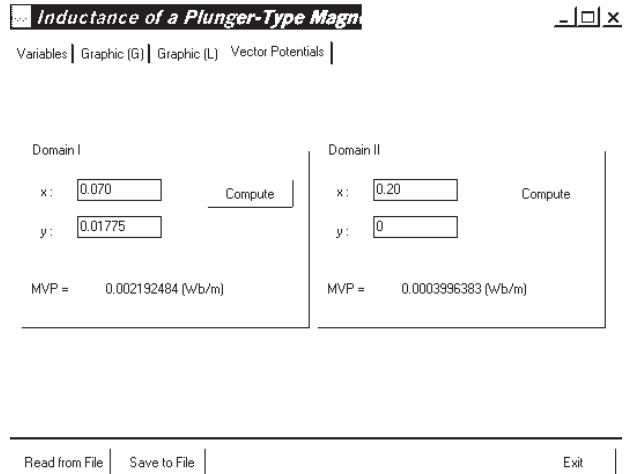

Fig. 11. The magnetic vector potential values

6 - CONCLUSIONS

1. The formula (16), considering the coil cross-section equal to window dimensions, gives the values of the internal partial inductance of plunger-type electromagnet which are in good agreement with the values obtained using FEM method (differences less than 4.5 %).

2. For total magnet inductance evaluation, two values of linear inductances can be considered: internal inductance L_{int}, which must be multiplied by the core thickness g and the external inductance L_{ext}, which must be multiplied by the rest of the mean turn of the coil length (27).

ACKNOWLEDGEMENT

REFERENCES

1. G. A. Cividjian et al., *Inductance of a plunger-type magnet*, IEEE Trans. on Magnetics, vol. 34, no. 5, Sept. 1998, pp. 3695-3688.
2. G. A. Cividjian et al., *Power Approximation of Boundary Conditions,* Seconde Conf. Europeenne sur les Methodes Numeriques en Electromagnetisme, Lyon, 19-21 Mars 1997, NUMELEC 97, 252-253.
3. H. Bateman, A. Erdelyi*, Higher Transcendental Function*, vol . 2, Mc. Graw-Hill, N. Y., 1953