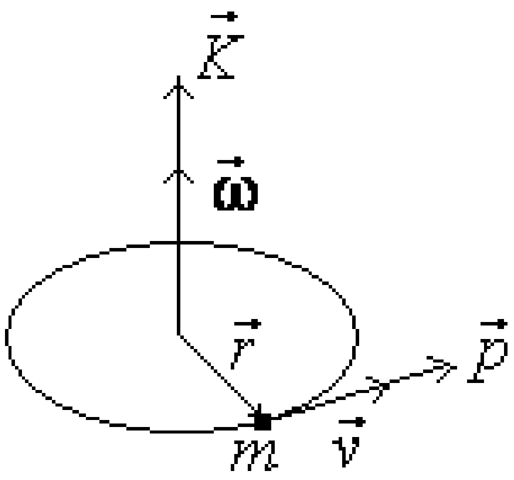
Mecanica

Impulsul: $\vec{p} = m\vec{v}$ $\qquad \dot{\vec{p}} = \vec{F}$ (Legea a II - a a lui Newton)

Momentul cinetic: $\vec{K} = \vec{r} \times \vec{p}$ $\qquad \dot{\vec{K}} = \vec{M}$ (Teorema momentului cinetic)



Momentul de inertie: $J = m r^2$;

Cuplul: $\vec{M} = \vec{r} \times \vec{F}$

Viteza periferica: $\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{\omega}$ – viteza unghiulara

$\vec{K} = J\,\vec{\omega}$

ξ = xi; ν = niu; ζ = zeta; κ = kappa; k = k; χ = hi

Legile electrotehnicii

	Legea	Forma locală (Maxwell)	Forma integrală	Notații şi u.m.
1	Circuitului magnetic (Ampère)	$\mathrm{rot}\vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$	$\oint_C \vec{H} \cdot \mathrm{d}\vec{l} = i$	H [A/m] câmp magnetic j [A/m^2] densitate de curent i [A] curent total
2	Inducției (Faraday, Lenz)	$\mathrm{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$e = -\frac{\partial \Psi}{\partial t}$	E [V/m] câmp electric e [V] t.e.m. Ψ [Wb] = [V.s] flux mag. total
3	Fluxului electric (Gauss)	$\mathrm{div}\vec{D} = \rho_v$	$\oiint_S \vec{D} \cdot d\vec{S} = q$	q [C] sarcina electrică D [C/m^2] inducția electrică ρ_v [C/m^3] dens. sarcină elec.
4	Fluxului magnetic	$\mathrm{div}\,\vec{B} = 0$	$\oiint_S \vec{B} \cdot d\vec{S} = 0$	B [T] = [Wb/m^2] inducția magnetică $\vec{B} = \mu\,\vec{H}$; $\vec{D} = \varepsilon\,\vec{E}$

Potențialele electrodinamice ale câmpului electromagnetic

Potențialul magnetic vector **A**:

$$\mathrm{rot}\,\mathbf{A} = \mathbf{B}$$

$$\left(\mathrm{div}\,\boldsymbol{A} = -\frac{1}{v}\frac{\partial U}{\partial t}\right)$$

Potențialul electrodinamic scalar *U*

$$\boldsymbol{E} + \frac{\partial \boldsymbol{A}}{\partial t} = -\mathrm{grad}\,U$$

Diferența de potențial: U_{AB}

$$U_{\mathrm{AB}} = \int_{\mathrm{A}}^{\mathrm{B}} \boldsymbol{E} \cdot \mathrm{d}\vec{l} + \frac{\partial \Psi}{\partial t}$$

Fluxul magnetic prin suprafața mărginita de curba C

$$\Psi = \iint_{\mathrm{S}} \mathbf{B} \cdot \mathrm{d}\vec{S} = \oint_{\mathrm{C}} \mathbf{A} \cdot \mathrm{d}\vec{l}$$

Unda electromagnetica

Ecuațiile lui d'Alambert

$$\Delta \boldsymbol{A} - \frac{1}{v^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu\, \boldsymbol{j} \qquad \Delta U - \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

Viteza de propagare a câmpului:

$$v = \frac{1}{\sqrt{\varepsilon\mu}}$$

In lipsa sarcinilor ρ si a curentilor $\boldsymbol{j}$ rezulta ecuatiile undelor:

$$\Delta \boldsymbol{A} = \frac{1}{v^2}\frac{\partial^2 \boldsymbol{A}}{\partial t^2} \qquad \Delta U = \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2}$$

Unda sferica. Potentiale retardate

$$\Delta U = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial U}{\partial r}\right) = \frac{1}{r}\frac{\partial^2 (rU)}{\partial r^2} \Rightarrow$$

$$\frac{\partial^2 (rU)}{\partial r^2} = \frac{1}{v^2}\frac{\partial^2 (rU)}{\partial t^2}$$

$$U = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho(t - r/v)}{r}\mathrm{d}V\,; \quad \boldsymbol{A} = \frac{\mu}{4\pi}\iiint_V \frac{\boldsymbol{j}(t - r/v)}{r}\mathrm{d}V$$

Legi de material

	Legea lui Ohm	$\vec{E} = \rho\,\vec{j};\ \ \vec{j} = \sigma\,\vec{E}$	$u + e = r\,i + \dfrac{q}{C}$ $U = Z \cdot I$	$\rho = 1/\sigma$ [Ω.m] C [F]; L [H]; $\Psi = L\,i$ $Z = \sqrt{r^2 + x^2}$ [Ω]
	Legea lui Joule	$p = \rho\,j^2 = \sigma\,E^2$	$P = r.I^2$	p [w/m^3] pierderile specifice

Legătura dintre inducție, câmp si polarizație (Materiale lineare)

$$\vec{D} = \varepsilon\,\vec{E} + \vec{P}; \qquad \varepsilon = \varepsilon_0\,\varepsilon_r; \quad \varepsilon_r = 1 + \chi_e$$

$$\vec{B} = \mu\,\vec{H} + \mu_0\,\vec{M}; \quad \mu = \mu_0\,\mu_r; \quad \mu_r = 1 + \chi_m$$

P – polarizația permanenta
M – magnetizația permanenta
χ_e, χ_m –susceptanțele el si mag

Regimurile fenomenelor electromagnetice

•**Staționare**: mărimi constante (c.c., j = ct.) - electrocinetica

Statice: $j = 0$ nu se transforma energia in căldura

electrostatica si magnetostatica

•**Variabile:** mărimi variabile in timp

Cvasistaționare: $\vec{j} >> \frac{\partial \vec{D}}{\partial t} \approx 0$ in afara de condensatoare

$\frac{\partial \vec{B}}{\partial t} \approx 0 \quad \left(\frac{\partial \vec{A}}{\partial t} \approx 0 \right)$ in dielectricul condensatoarelor

Unde electromagnetice: $j = 0, \quad \rho = 0$

Conectarea unui circuit RC la tensiune continua

Teorema comutației: ψ(-0) = ψ(+0) si $q(-0) = q(+0)$

$$\tau_c = RC$$

$$u_c(0) = 0$$

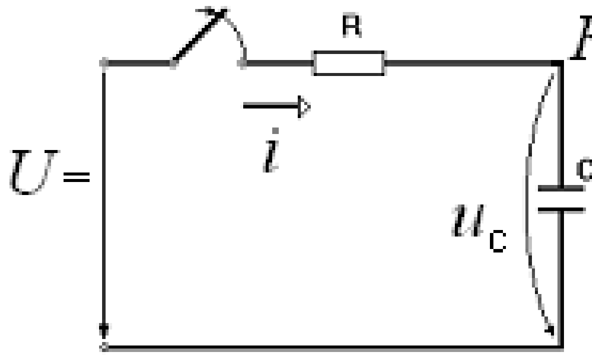


Fig. 6

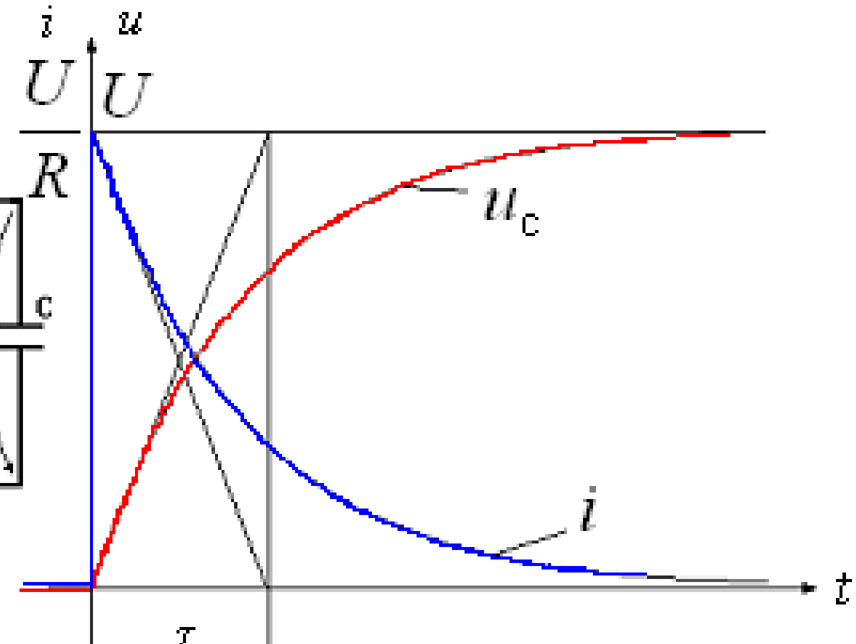


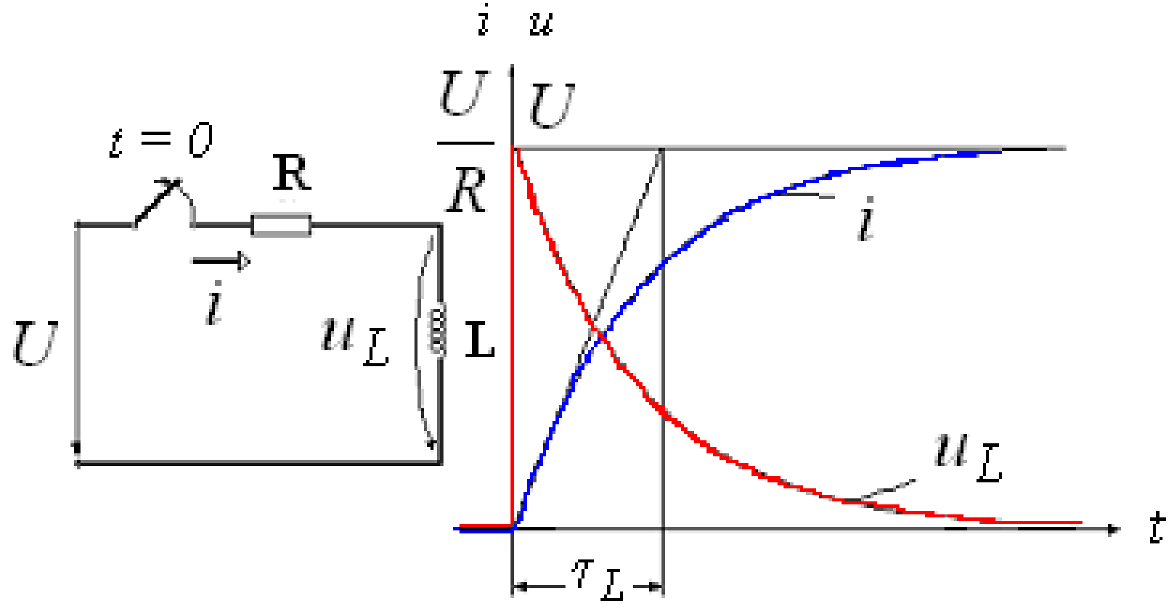
Fig. 7

$$i(t) = \frac{U}{R} e^{-t/\tau_c}; \qquad u_c(t) = U\left[1 - e^{-t/\tau_c}\right]$$

Conectarea unui circuit R-L la tensiune continua

Teorema comutației: $\psi(-0) = \psi(+0)$ si $q(-0) = q(+0)$

$$\tau_L = \frac{L}{R}$$



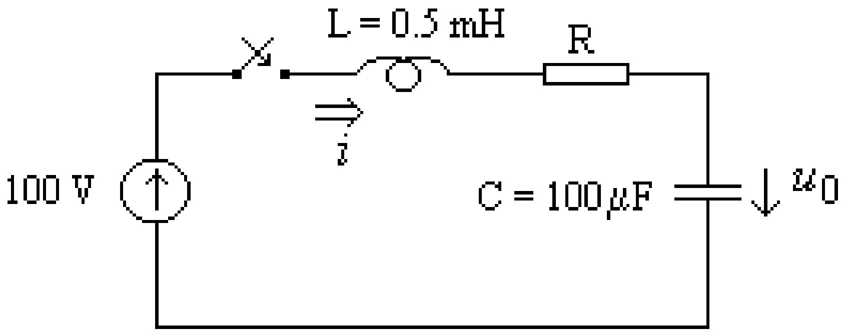
$$u_L(t) = U e^{\frac{-t}{\tau_L}} \qquad i(t) = \frac{U}{R}\left[1 - e^{\frac{-t}{\tau_L}}\right]$$

Conectarea unui circuit RLC la tensiune continua

Teorema comutației: $\psi(-0) = \psi(+0)$ et $q(-0) = q(+0)$

$$\delta \overset{n}{=} \frac{R}{2L}; \quad \omega_0^2 \overset{n}{=} \frac{1}{LC}; \quad \omega_1^2 \overset{n}{=} \omega_0^2 - \delta^2; \quad tg\theta \overset{n}{=} \frac{\delta}{\omega_1} \quad R_c = 2\sqrt{\frac{L}{C}}$$

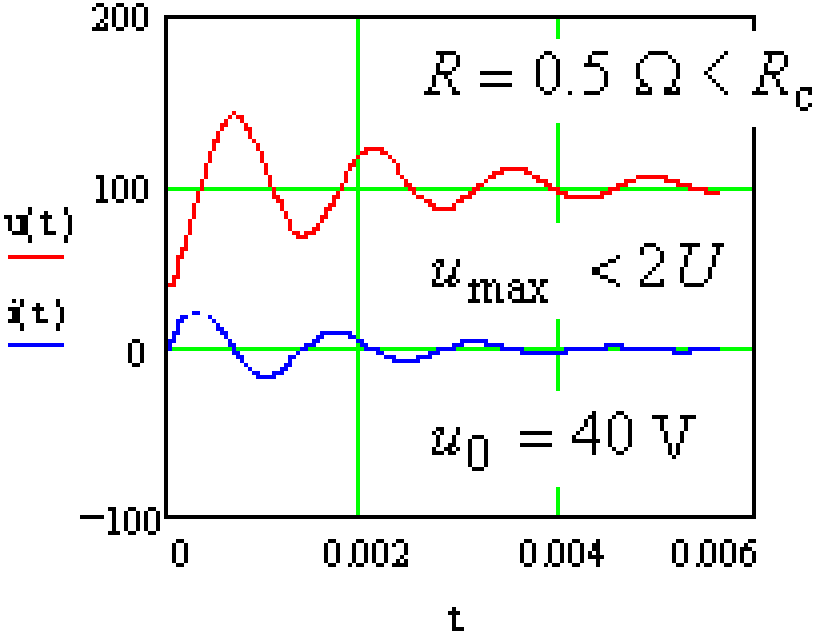
$$u(t) = (U - u_0)\left[1 - e^{-\delta t}\,\frac{\cos(\omega_1 t - \theta)}{\cos\theta}\right] + u_0$$

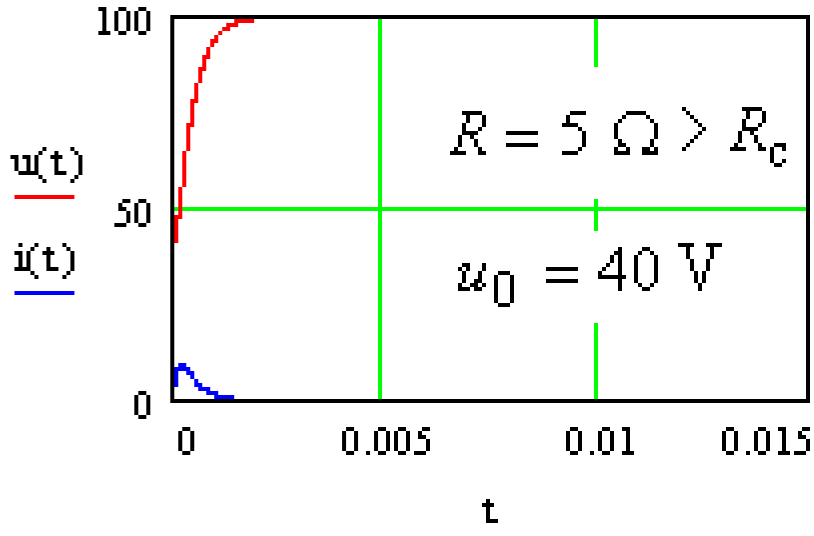


$$i(t) = \hat{I} e^{-\delta t} \sin\omega_1 t$$

$$\hat{I} = \frac{U - u_0}{\omega_1 L}$$

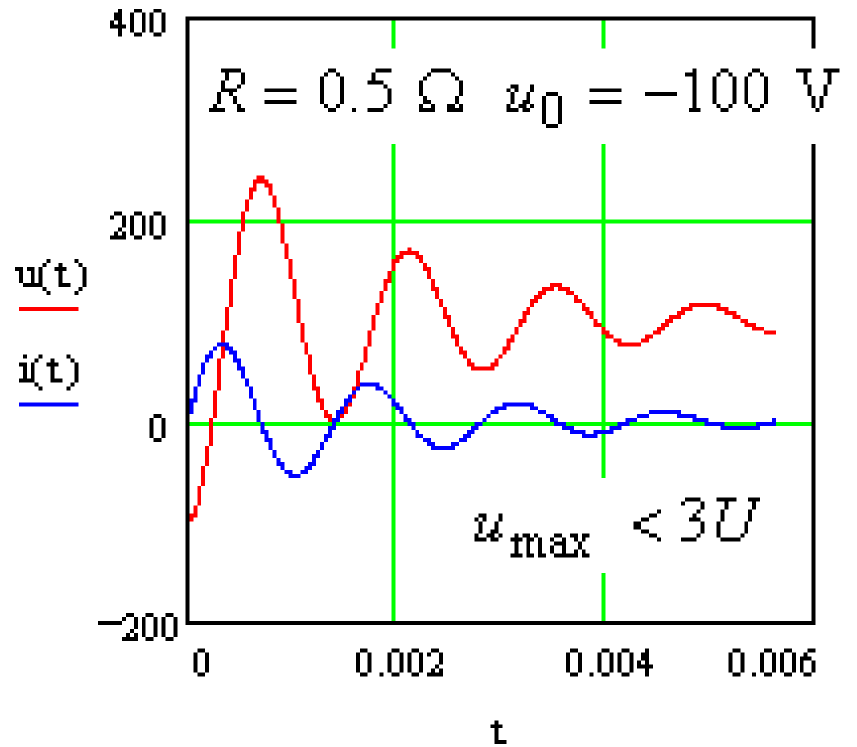
U=100 V
L=0.5 mH C= 100 μF



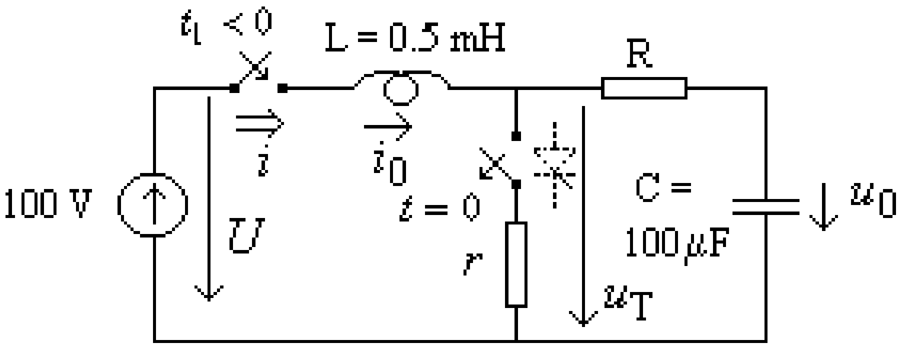


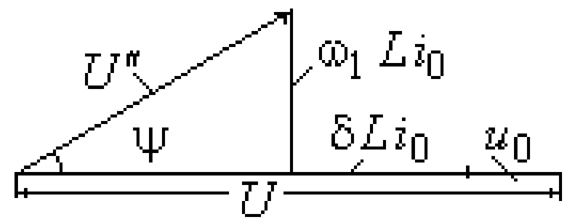
Exemple

$R_c = 4.5\ \Omega \qquad f_1 = 707\ \text{Hz}$



Blocarea thyristorului



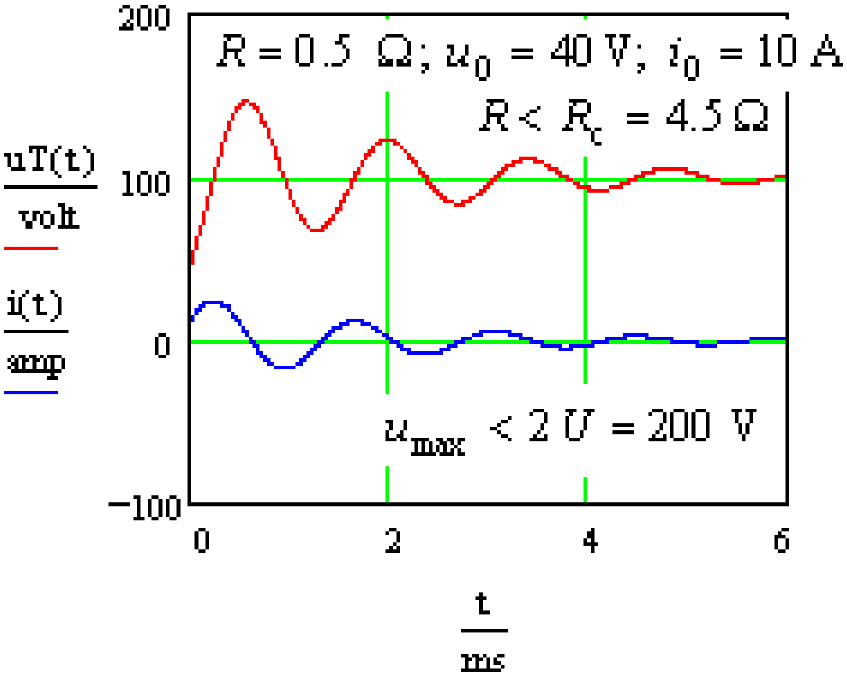


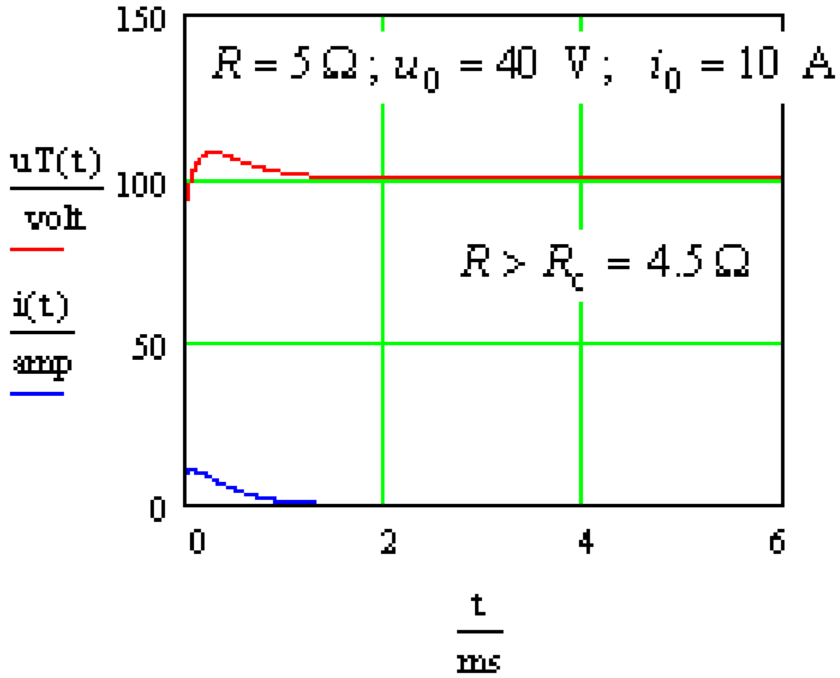
$$I = \frac{U''}{\omega_1 L}; \quad U'' = \sqrt{(U - u_0 - \delta L i_0)^2 + (\omega_1 L i_0)^2}; \quad \tan\psi = \frac{\omega_1 L i_0}{U - u_0 - \delta L i_0};$$

$$\tan\theta = \frac{\delta}{\omega_1}$$

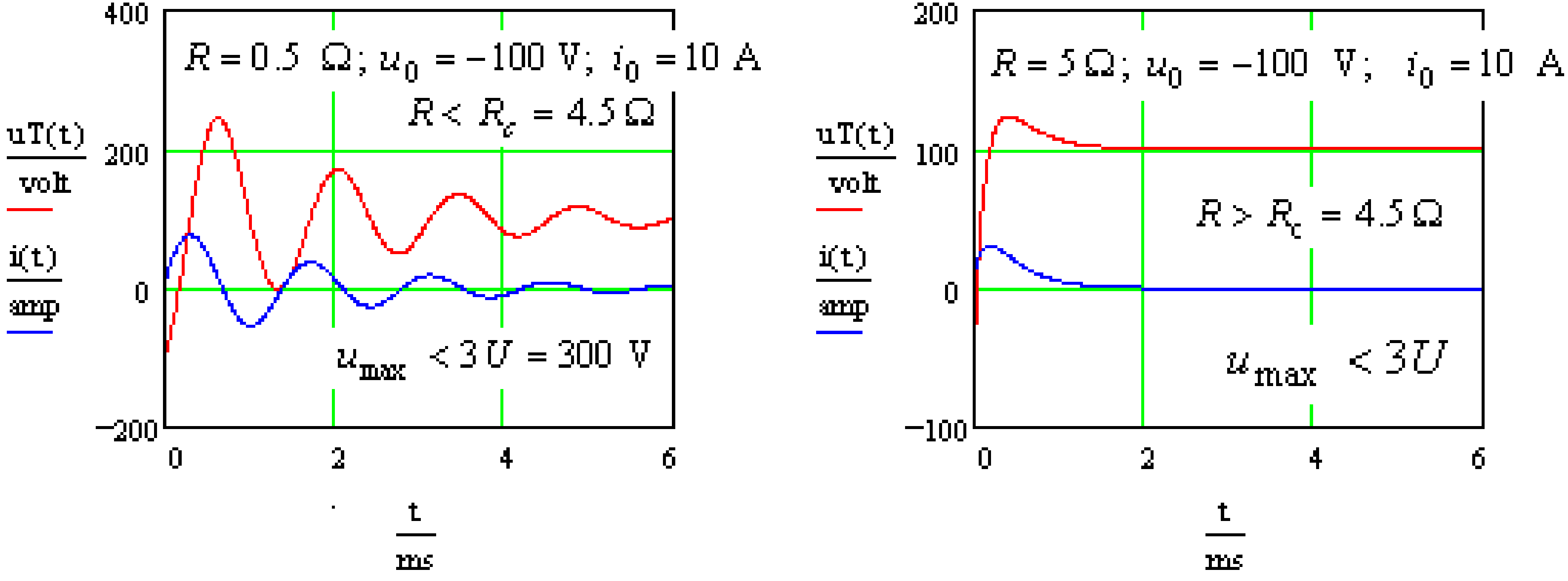
$$i(t) = I e^{-\delta t} \sin(\omega_1 t + \psi); \quad u_T(t) = U - U'' e^{-\delta t} \frac{\cos(\omega_1 t + \psi + \theta)}{\cos\theta}$$

Regimul tranzitoriu oscilant si aperiodic la $u_0 = +40$ V

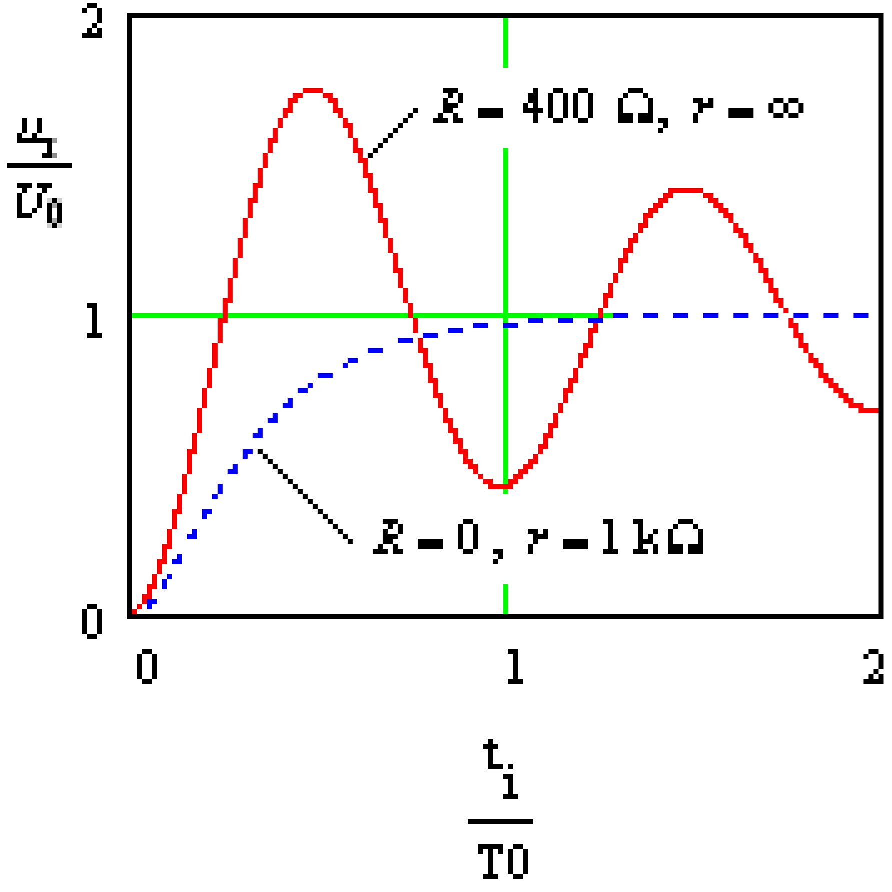


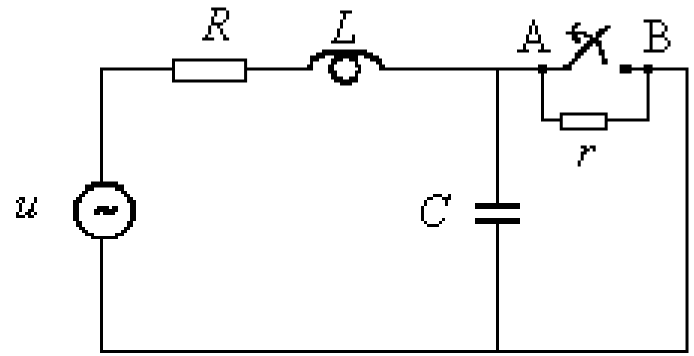


Regimul tranzitoriu oscilant si aperiodic la u_0 = -100 V



Exemplu: TTR la intreruptor suntat





$$L = 95 \text{ mH} \qquad C = 20000 \text{ pF}$$

$$r_{\text{cr}} = \frac{z_{\text{c}}}{2 + R / z_{\text{c}}}; \quad z_{\text{c}} = \sqrt{\frac{L}{C}}$$

$$R_{\text{cr}} = 2\, z_{\text{c}}$$

$R := 400 \cdot ohm \qquad L := 95 \cdot mH \qquad C := 20000 \cdot pF$

$R_{cr} := 2 \cdot \sqrt{\frac{L}{C}} \qquad R_{cr} = 4.359 \cdot 10^3 \cdot ohm \qquad \omega 0 := \frac{1}{\sqrt{L \cdot C}} \qquad \omega 0 = 2.294 \cdot 10^4 \cdot sec^{-1}$

$r_{cr} := \frac{1}{2} \cdot \sqrt{\frac{L}{C}} \qquad r_{cr} = 1.09 \cdot 10^3 \cdot ohm \qquad f_0 := \frac{\omega 0}{2 \cdot \pi} \qquad f_0 = 3.651 \cdot 10^3 \cdot sec^{-1}$

$\delta := \frac{R}{2 \cdot L} \qquad \delta = 2.105 \cdot 10^3 \cdot sec^{-1} \qquad \gamma := 1 + \exp\left(-\frac{\delta}{2 \cdot f_0}\right) \qquad \gamma = 1.75$

$R := 0 \cdot ohm \qquad r := 1000 \cdot ohm$

$\delta 1 := \frac{1}{2} \cdot \left(\frac{R}{L} + \frac{1}{r \cdot C}\right) \qquad \delta 1 = 2.5 \cdot 10^4 \cdot sec^{-1} \qquad \omega 1 := \sqrt{\omega 0^2 \cdot \left(1 + \frac{R}{r}\right) - \delta 1^2}$

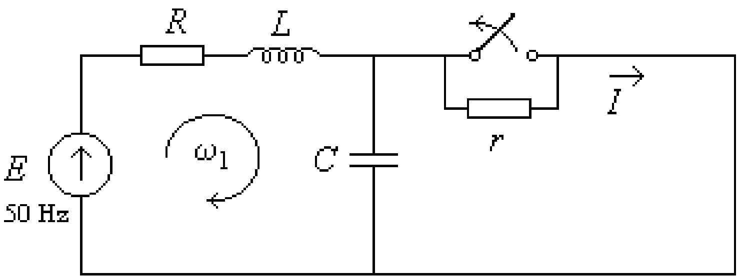
$u(t) := 1 - e^{-\delta \cdot t} \cdot \cos(\omega 0 \cdot t) \qquad i := 0 .. 1000 \qquad T0 := \frac{1}{f_0} \qquad t_i := \left(2 \cdot T0 \cdot \frac{i}{1000}\right)$

$u_1(t) := \frac{r}{R + r} \cdot \left[1 - e^{-\delta 1 \cdot t} \cdot \left(\cos(\omega 1 \cdot t) + \frac{\delta 1}{\omega 1} \cdot \sin(\omega 1 \cdot t)\right)\right]$

$\omega 1 = 9.934 \cdot 10^3 i \cdot sec^{-1}$

comutație

TTR la întreruptor şuntat



$x=\omega L=30\ \Omega$

$R=400\ \Omega$

$C=20000$ pF

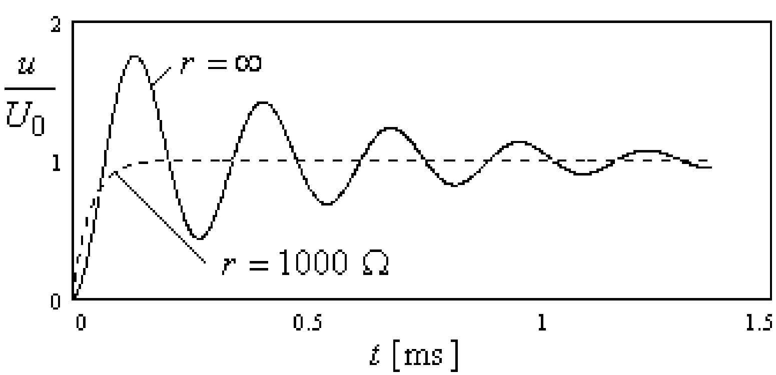
$r=\begin{cases}\infty\\ 1000\ \Omega\end{cases}$

$$R_{\text{cr}}=2\sqrt{\frac{L}{C}}=2\sqrt{\frac{0.095}{20000\cdot10^{-12}}}=4360\ \Omega;$$

$$r_{\text{cr}}=\frac{1}{2\sqrt{\frac{C}{L}}+R\frac{C}{L}}=\frac{1}{2\sqrt{\frac{2\cdot10^{-8}}{0.095}}+400\frac{2\cdot10^{-8}}{0.095}}=998$$

$$f_0=\frac{1}{2\pi\sqrt{LC}}=3261\quad \delta=\frac{R}{2L}=2105$$

$$\delta_s=\frac{1}{2}\left(\frac{R}{L}+\frac{1}{rC}\right)=\frac{1}{2}\left(\frac{400}{0.095}+\frac{1}{1000\cdot2\cdot10^{-8}}\right)=27110$$

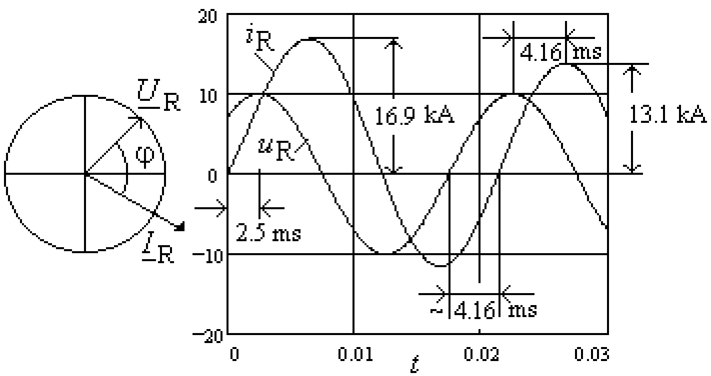


$$f_1=\sqrt{f_0^2\left(1+\frac{R}{r}\right)-\left(\frac{\delta_s}{2\pi}\right)^2}=233\text{ Hz}$$

$$\gamma=1+e^{-\frac{\delta}{2f_0}}=1.75\qquad \gamma\big|_{r=1000}\approx1$$

$$u(t)=U_0\left(1-e^{-\delta t}\cos(\omega_1 t)\right)$$

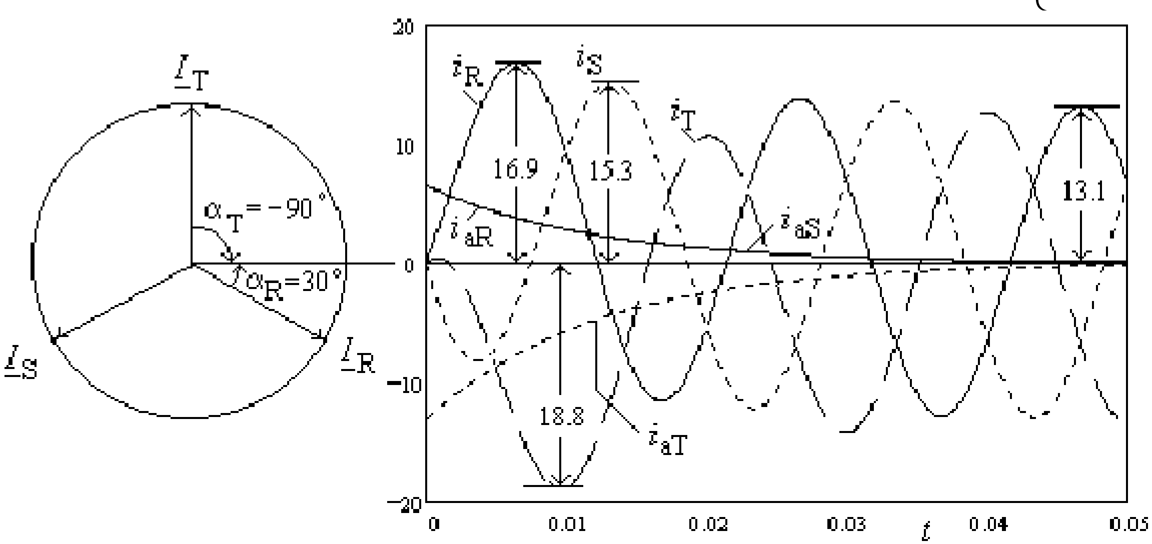
Scurt-circuit trifazat simetric



$$u_R = U\sin(\omega t - \psi_R); \qquad \psi_R = \frac{2.5-5}{10}180° = -45°$$

$$\varphi = \frac{4.16}{10}180 = 75° \qquad \tau = \frac{\tan\varphi}{\omega} = \frac{3.73}{100\,\pi} = 0.012\ \text{s}$$

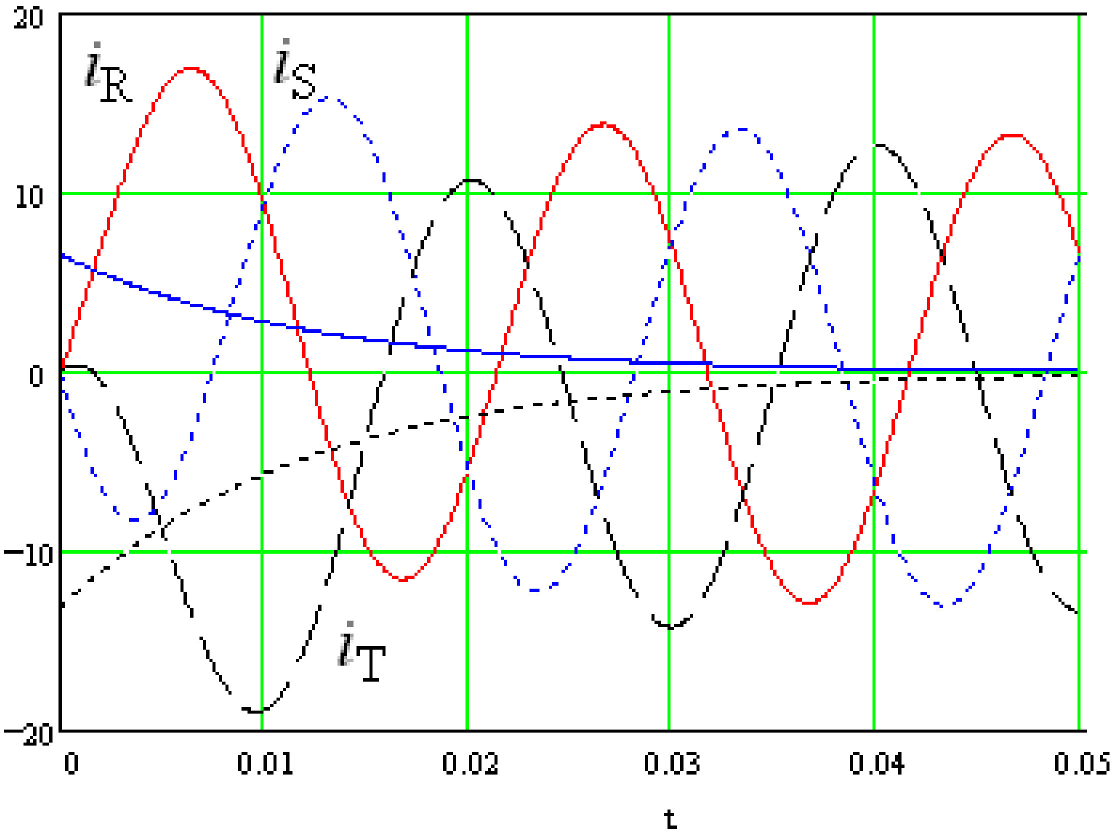
$$\alpha = \psi + \varphi = \begin{cases} 30° \\ 150° \\ -90° \end{cases} \qquad \omega t_m = \alpha + \frac{\pi}{2}\left(1 - 2\,\text{Int}\left(\frac{\alpha}{\pi}\right)\right) = \begin{cases} 120° \\ 240° \\ 180° \end{cases}$$



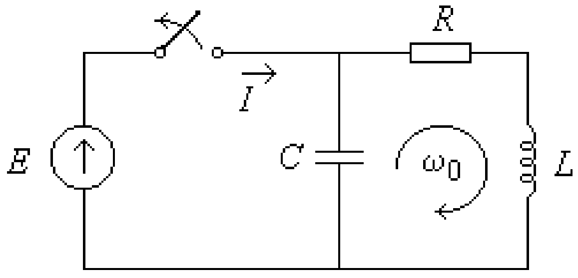
$$i_y = \hat{I}_\infty\left[\sin(\omega t_m - \alpha) + \sin\alpha\, e^{-\frac{t_m}{\tau}}\right] = \begin{cases} 16.9 \\ 15.3 \\ -18.8 \end{cases} \text{[kA}$$

$$I_T(0.010) = 13.15\sqrt{\frac{1}{2} + e^{-\frac{2\cdot 0.01}{0.012}}} = 10.9\ \text{[kA]}$$

Scurtcircuit trifazat simetric



Supratensiunea la deconectarea unui curent continuu



$I = 5$ A

$L = 1.5$ H

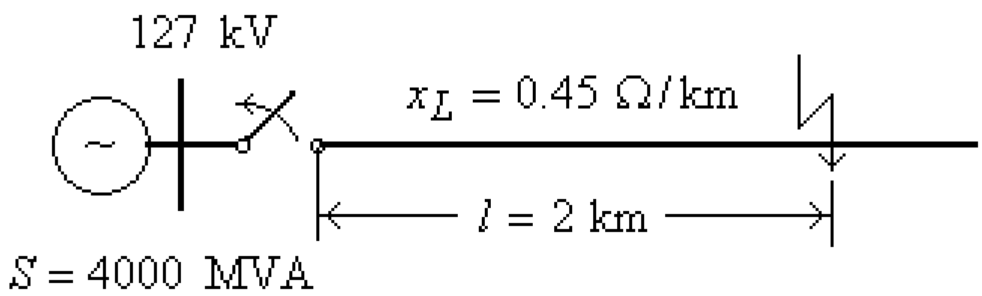
$C = 0.1$ μF

$R \approx 0$ Ω

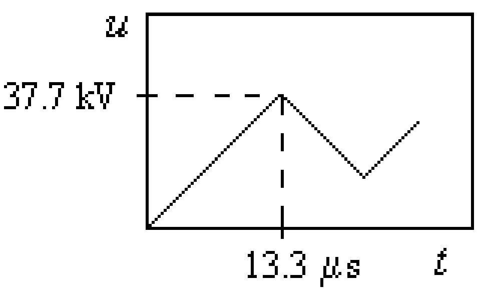
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1.5\cdot 10^{-5}}} = 411 \text{ Hz}$$

$$\frac{C u_{max}^2}{2} = \frac{L I^2}{2} \quad \Rightarrow \quad u_{max} = I\sqrt{\frac{L}{C}} = 5\sqrt{\frac{1.5}{0.1\cdot 10^{-6}}} = 19365 \text{ V}$$

TTR la defect kilometric



a)



b)

$$x_1 = \frac{U^2}{S} = \frac{127^2}{4000} = 4.032\ \ \Omega$$

$$I = \frac{U}{\sqrt{3}\,(x_1 + x_2)} = \frac{127}{\sqrt{3}\,(4.032 + 0.9)} = 14.87\ \ \mathrm{kA}$$

$$x_2 = l\,x_L = 2 \cdot 0.45 = 0.9\ \Omega$$

$$u_1 = 1.27\,I\,l = 1.27 \cdot 14.87 \cdot 2 = 37.76\ \ \mathrm{kV}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 0.22\,I = 0.22 \cdot 14.87 = 3.27\ \ \frac{\mathrm{kV}}{\mu\mathrm{s}}$$

$$t_1 = \frac{2l}{c} = \frac{2 \cdot 2}{0.3} = 13.3\ \ \mu\mathrm{s}$$

Reabilitarea termica a apartamentului cu pereti de beton prefabricat (3 camere)

Suprafata peretilor: $Snet := 50 \cdot m^2$

Grosimea peretilor $d_{ext} := 7 \cdot cm$ $d_{int} := 12 \cdot cm$

Supraf. ferestrelor: $Sf := 9.775 \cdot m^2$

$\lambda_{2600} := 2.03 \cdot \frac{watt}{m \cdot K}$ $d_{vata} := 8 \cdot cm$

Rez. termica a 1 m^2 de fereastra: $Rf := 0.37 \cdot m^2 \cdot \frac{K}{watt}$ $\lambda_{vataMin} := 0.07 \cdot \frac{watt}{m \cdot K}$

Coef. de cedare a caldurii: $\alpha_{int} := 6 \cdot \frac{watt}{m^2 \cdot K}$ $\alpha_{ext} := 23 \cdot \frac{watt}{m^2 \cdot K}$

Grosimea izolatiei de polisteren: $d := \begin{bmatrix} 4 \\ 5 \\ 8 \\ 10 \\ 12 \end{bmatrix} \cdot cm$

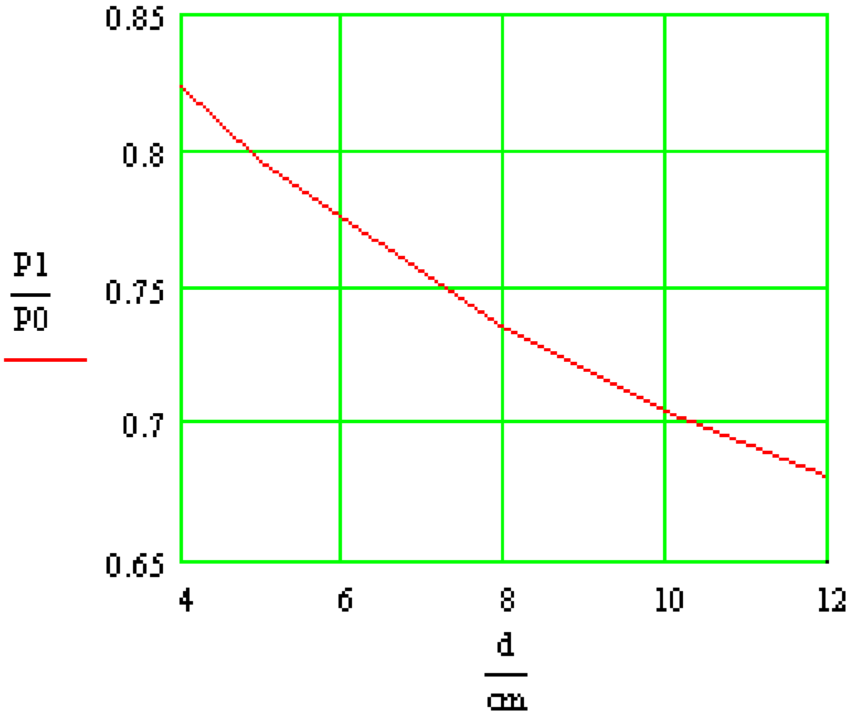
Scaderea pierderilor prin pereti in functie de grosimea izolatiei

$$P0 := \Delta T \cdot \left(\frac{Sf}{Rf} + \frac{Snet}{R0} \right) \qquad \Delta T := 40 \cdot K$$

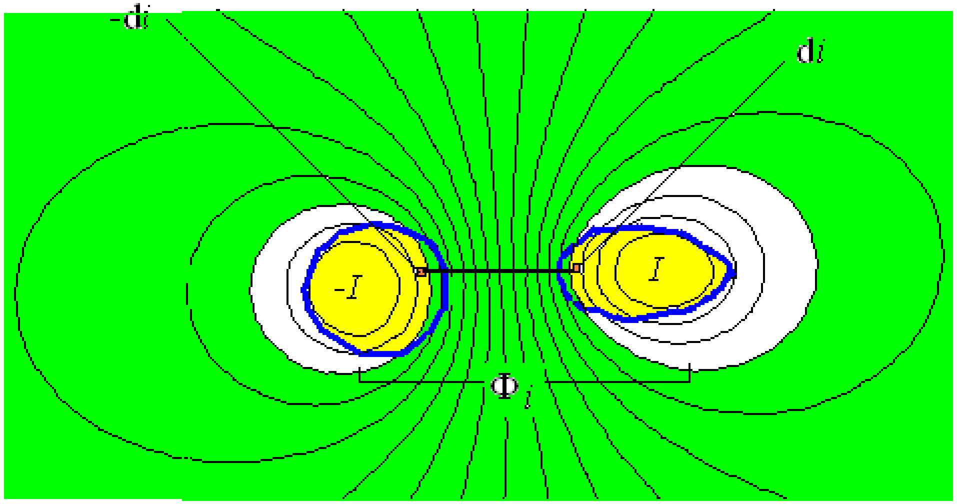
$$P1 := \overrightarrow{\Delta T \cdot \left(\frac{Sf}{Rf} + \frac{Snet}{Riz} \right)}$$

$$P0 = 2.278 \cdot kW$$

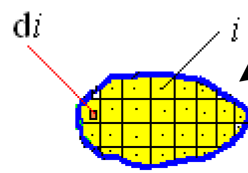
$$\frac{P1}{P0} = \begin{bmatrix} 0.824 \\ 0.797 \\ 0.735 \\ 0.705 \\ 0.681 \end{bmatrix} \qquad P1 = \begin{bmatrix} 1.877 \\ 1.815 \\ 1.675 \\ 1.607 \\ 1.552 \end{bmatrix} \cdot kW$$



Fluxul total
(înlanțuirea magnetica cu *i*)



Fluxul fascicular îmbrățişat de firul de curent di



$$I = wi$$

$$\Psi = \frac{1}{i}\int_I \Phi_i \, \mathrm{d}i$$

$$\Psi = \frac{1}{i}\int_\Phi i_\Phi \, \mathrm{d}\Phi$$

Curentul îmbrățişat de tubul de flux dΦ