Impedance voltage of power multi-winding autotransformer

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Abstract: In the paper a leakage magnetic field of large power multi-winding autotransformer with regulating windings is analyzed using a 2D finite element method. It is shown that, for impedance voltage determination for all the pairs of windings and all the regulating windings tapping, the cylindrical model of windings can be used, considering the symmetry axis of the transformer window as a magnetic flux line.

The International Electrotechnical Vocabulary [1] define the impedance voltage of a multi-winding transformer for the principal tapping, related to a certain pair of windings as the voltage, required to be applied at rated frequency to the line terminals of one of the windings of a pair for a polyphase transformer, or to the terminals of such a winding for a single-phase transformer, to cause a current to flow through these terminals corresponding to the smaller of the rated power values of both windings of the pair, the terminals of the other winding of the pair being short-circuited and the remaining windings being open-circuited. The various values for the different pairs are normally related to the appropriate reference temperature. The impedance voltage at rated current is usually expressed as a percentage of the rated voltage of the winding to which the voltage is applied. The short-circuit impedance of a pair of windings is defined, in the same vocabulary, as the equivalent star connection impedance, related to one of the windings, for given tapping and expressed in ohms per phase, at rated frequency, measured between the terminals of a winding when the other winding is short-circuited (the value is normally related to the appropriate reference temperature).

If in the case of principal tapping the impedance voltage of power transformers can be easy evaluated using the simple formulas, considering straight magnetic flux lines [2], in the case of other tapping of regulating winding consideration must be given to the magnetic field pattern.

Basic equations

Let be w_1, w_2, w_3 and w_R the number of turns of the three winding three-phase power autotransformer with regulating winding having respectively the powers S_1, S_2 and S_3 in MVA. The one-column powers will be:

$$S'_i = S_i / 3, \quad i = 1, 2, 3 \tag{1}$$

Primary and secondary winding

The secondary winding is connected to the median (principal) tapping of the regulating winding, so that the phase secondary voltages for median and marginal tapping will be respectively given by the vector:

$$\vec{U}_2 = U_{2\text{phr}} \cdot \begin{bmatrix} 1 & 1+\dfrac{w_\text{R}}{2\,w_2} & 1-\dfrac{w_\text{R}}{2\,w_2} \end{bmatrix}; \qquad U_{2\text{phr}} = \frac{U_1}{\sqrt{3}} \frac{w_2}{w_1 + w_2} \tag{2}$$

Here U_1 is the primary rating voltage.

For the determination of the short-circuit parameters we will consider the system linear one and consequently the magnetic field energy can be calculated for any arbitrary line value of the primary current $I_{1\text{e}}$ (usually close to the primary rating current $I_{1\text{r}}$). For star connection the phase current will be the same and the corresponding secondary tapping currents result from the equality of the primary and secondary ampere-turns and will be given by the vector:

$$I_{1\text{eph}} = I_{1\text{e}}; \qquad \vec{I}_2 = I_{1\text{eph}} \frac{w_1}{w_2} \cdot \begin{bmatrix} 1 & \dfrac{1}{1+\dfrac{w_R}{2\,w_2}} & \dfrac{1}{1-\dfrac{w_R}{2\,w_2}} \end{bmatrix} \tag{3}$$

The primary, secondary and regulating winding total cross-sections A_1, A_2 and A_R determine the corresponding current densities:

$$j_1 = w_1 \frac{I_{1\text{eph}}}{A_1}; \qquad \vec{j}_2 = w_2 \frac{\vec{I}_2}{A_2}; \qquad \vec{j}_\text{R} = w_\text{R} \frac{\vec{I}_2}{A_\text{R}} \tag{4}$$

Usually the primary and regulating windings are formed by two parallel-connected coils. In this case, in the above equations, the number of turn of one coil and the total cross-section of both coils must be considered.

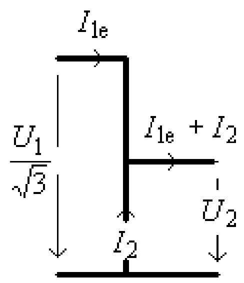


Fig. 1.Autotransformer schema

Let be W_{12} the vector of magnetic field energy, produced by the currents $I_{1\text{e}}$ and I_2, evaluated using the finite element method. The referred to primary winding short-circuit reactance will be:

$$\vec{x}_{\text{k}12} = \omega \frac{2\vec{W}_{12}}{I_{1\text{e}}^2} \ [\Omega]; \qquad \vec{x}_{\text{k}12\%} = 100 \frac{\vec{x}_{\text{k}12}}{x_{1\text{r}}} \ [\%]; \qquad x_{1\text{r}} = \frac{U_{1\text{r}}}{\sqrt{3}\, I_{1\text{r}}} \tag{5}$$

Primary and tertiary winding

Usually the tertiary winding rating power is smaller, so the short-circuit reactance will be referred to the tertiary winding rated line voltage $U_{3\text{r}}$ and line current $I_{3\text{r}}$. This is also preferable if the tertiary winding has not several tapping. In this case the equivalent star connection rated impedance will be similar with the above:

$$x_{3\text{r}} = \frac{U_{3\text{r}}}{\sqrt{3}\, I_{3\text{r}}} \tag{6}$$

For delta connection of the tertiary winding, the phase currents for energy calculation are:

$$I_{3\text{eph}} = \frac{I_{3\text{e}}}{\sqrt{3}}; \qquad \vec{I}_{1\text{c}} = w_3 I_{3\text{eph}} \cdot \left[\frac{1}{w_1 + w_2} \quad \frac{1}{w_1 + w_2 + \frac{w_\text{R}}{2}} \quad \frac{1}{w_1 + w_2 - \frac{w_\text{R}}{2}} \right] \tag{7}$$

The corresponding current densities are:

$$\vec{j}_1 = w_1 \frac{\vec{I}_{1\text{c}}}{A_1}; \quad \vec{j}_2 = w_2 \frac{\vec{I}_{1\text{c}}}{A_2}; \quad j_3 = w_3 \frac{I_{3\text{eph}}}{A_3}; \quad \vec{j}_\text{R} = w_\text{R} \frac{\vec{I}_{1\text{c}}}{A_\text{R}} \tag{8}$$

Let be W_{13} the vector of magnetic field energy, produced by the currents $I_{3\text{e}}$ and $I_{1\text{c}}$, evaluated using the finite element method. The referred to tertiary winding (equivalent to star connection) short-circuit reactance will be:

$$\vec{x}_{\text{k}13} = \omega \frac{2\vec{W}_{13}}{I_{3\text{e}}^2} \ [\Omega]; \qquad \vec{x}_{\text{k}13\%} = 100 \frac{\vec{x}_{\text{k}13}}{x_{3\text{r}}} \ [\%]; \qquad x_{3\text{r}} = \frac{U_{3\text{r}}}{\sqrt{3}\, I_{3\text{r}}} \tag{9}$$

Secondary and tertiary winding

The determination of the short-circuit reactance for the secondary and tertiary windings is similar with the previous case. The secondary and regulating winding currents will be:

$$I_{3\text{eph}} = \frac{I_{3\text{e}}}{\sqrt{3}}; \qquad \vec{I}_{2\text{c}} = w_3 I_{3\text{eph}} \cdot \left[\frac{1}{w_2} \quad \frac{1}{w_2 + \frac{w_\text{R}}{2}} \quad \frac{1}{w_2 - \frac{w_\text{R}}{2}} \right] \tag{10}$$

The current densities:

$$\vec{j}_2 = w_2 \frac{\vec{I}_{2\text{c}}}{A_2}; \quad j_3 = w_3 \frac{I_{3\text{eph}}}{A_3}; \quad \vec{j}_\text{R} = w_\text{R} \frac{\vec{I}_{2\text{c}}}{A_\text{R}} \tag{11}$$

Let be W_{23} the vector of magnetic field energy, produced by the currents $I_{3\text{e}}$ and $I_{2\text{c}}$, evaluated using the finite element method. The referred to tertiary winding (equivalent to star connection) short-circuit reactance will be:

$$\vec{x}_{\text{k}23} = \omega \frac{2\vec{W}_{23}}{I_{3\text{e}}^2} \ [\Omega]; \qquad \vec{x}_{\text{k}23\%} = 100 \frac{\vec{x}_{\text{k}23}}{x_{3\text{r}}} \ [\%]; \qquad x_{3\text{r}} = \frac{U_{3\text{r}}}{\sqrt{3}\, I_{3\text{r}}} \tag{12}$$

Magnetic energy evaluation

The magnetic field problem will be solved using Femme 3.1 program [4] for static magnetic field and axisymmetric solution.

We will assume a cylindrical symmetry of the windings and of the magnetic field and, due to the relatively small size of the coil conductors, will neglect the skin effect in the conductors and in the screens. Consequently, the coil conductivity will be taken equal to zero and the frequency also will be considered zero.

Boundary conditions

In the transformer window the three sides with ferromagnetic borders a zero tangential component of the magnetic field will be considered. For frontal part of the coils, on the right side vertical border, a small skin depth should be considered because of aluminum or copper screen of the transformer tank. But, for commercial frequency, the magnetic field determined in these conditions is practically coincident with the static field determined for frequency equal to zero and considering the right side border as zero magnetic potential line. The following figures show that this approximation (A = 0) can be enough well applied also for the (a-b) axis of transformer window. Indeed, when the current in T phase reach its maximum, the currents in R and S are equal and the field pattern is completely symmetrical. In fig. 3 and 4 the vector magnetic potential variations, along the symmetry axis between the two phases and along the bottom window border, are given for maximum current in phase R and in fig. 6 and 7 for zero current in phase R. The very small values of the magnetic potential module on the axis can be observed. The approximation is also confirmed by the practical equality of the magnetic field energies, calculated in two cases (for A = 0 border, the energy per m results with 0.5% smaller).

The introduction of this border has the advantage of the possibility to apply a cylindrical model and of the two times reduction of the nodes number.

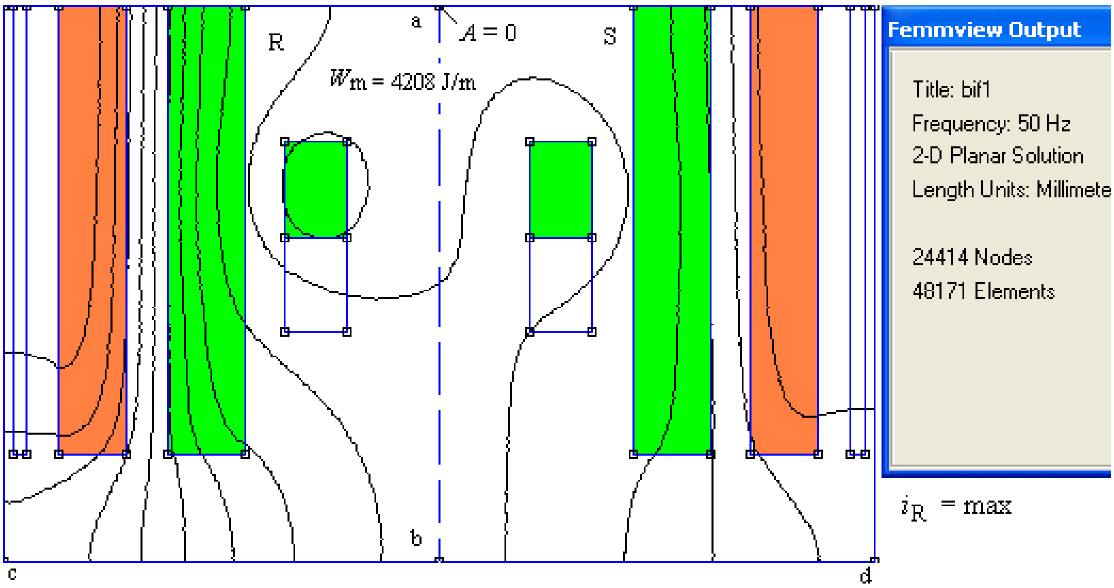


Fig. 2. Magnetic field pattern for the transformer window with 2 phases, at minus tapping for i_R = max

Phase	R	S
j_1 [A/mm^2]	0.8	-0.4-0.692j
j_2 [A/mm^2]	-1.198	0.6+1.037j
j_R [A/mm^2]	1.392	-0.696-1.205j

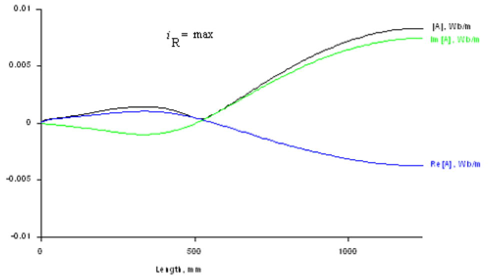


Fig. 3 Vector magnetic potential along the symmetry axis (a-b) of the transformer window

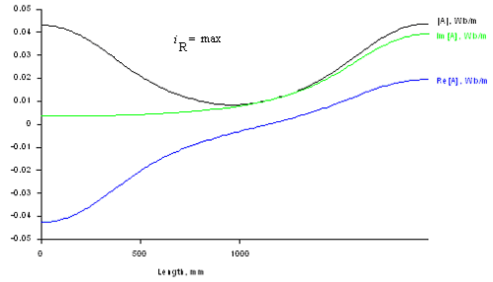


Fig. 4. Vector magnetic potential along the bottom border (c-d) of the transformer window for $i_R = \max$

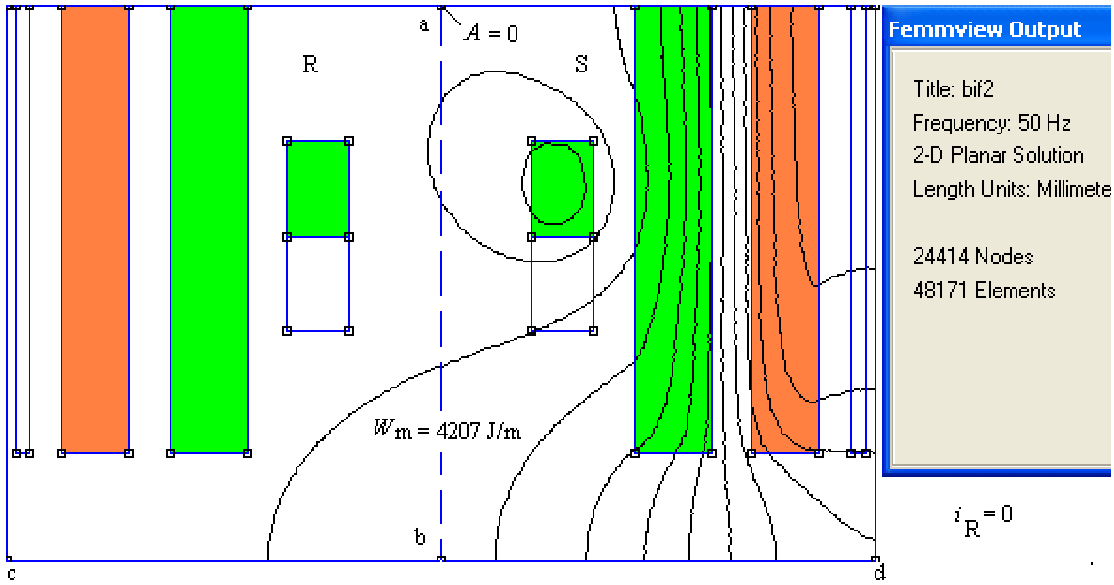


Fig. 5. Magnetic field pattern for the transformer window with 2 phases, at minus tapping for $i_R = 0$

Phase	R	S
j_1 [A/mm^2]	0.8 j	0.693-0.4 j
j_2 [A/mm^2]	-1.198 j	-1.037+0.6 j
j_R [A/mm^2]	1.392 j	1.206-0.696 j

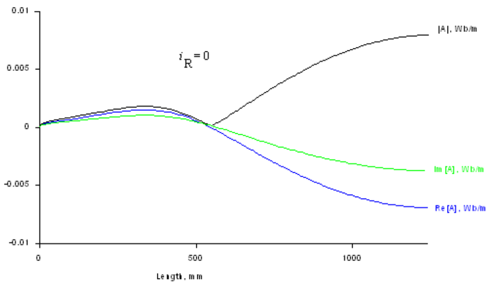


Fig. 6. Vector magnetic potential along the symmetry axis (a-b) of the transformer window

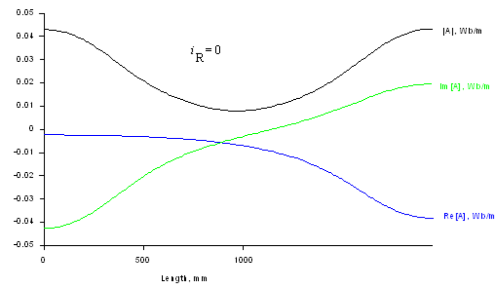
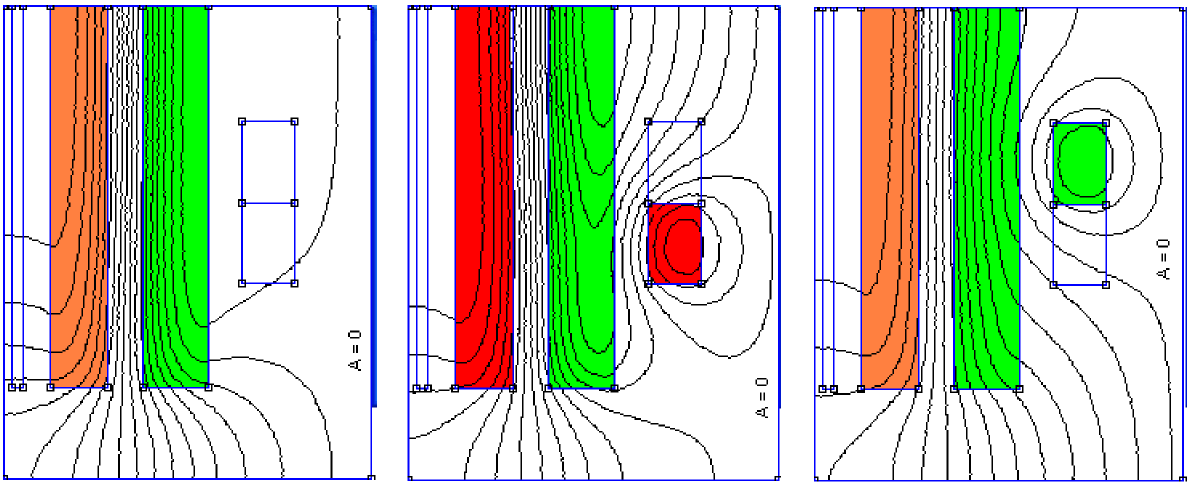


Fig. 7. Vector magnetic potential along the bottom border (c-d) of the transformer window for $i_R = 0$

Example

In the following figures the magnetic field patterns are given for each pairs of the windings of the 400/400/80 MVA power autotransformer for 400/231/22 kV. In the next table the apparent current densities, magnetic field energies and the values of short-circuit reactance are given for principal, plus and minus tapping. The values are compared with the corresponding experimental values of the impedance voltage.



Principal tapping | Plus tapping | Minus tapping

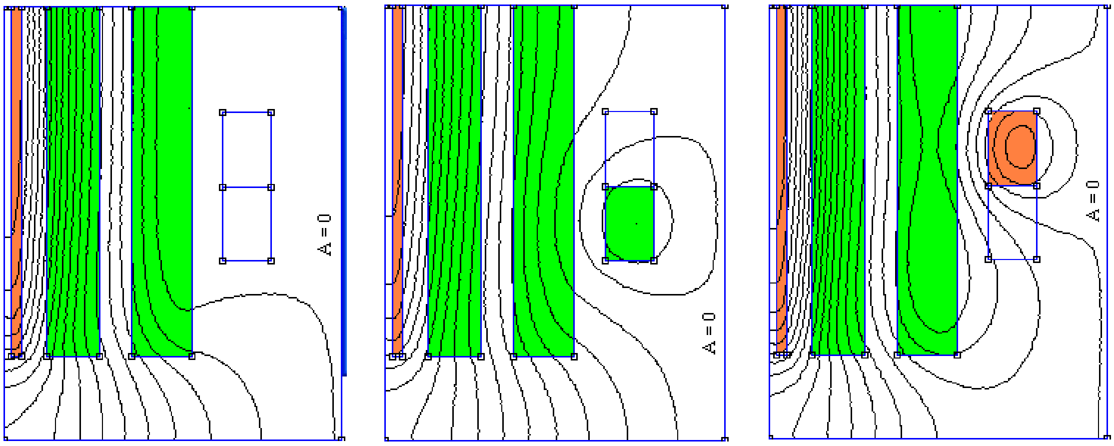
Fig. 8. Magnetic field in inferior half window of the autotransformer for primary-secondary windings short-circuit test

Primary and secondary windings

I_{1e} [A]	j_1 [A/mm^2]	j_2 [A/mm^2]	j_R [A/mm^2]	W_{12} [J]	x_{k12} [Ω]	x_{k12} [%]	x_{k12} [Ω] [exp.]	Error [%]
577.1	0.8	-0.921	0	10883	41.06	10.26	40.41	-1.6
577.1	0.8	-0.749	-0.868	7657	28.89	7.22	29.56	2.3
577.1	0.8	-1.198	1.389	21974	82.91	20.73	82.68	-0.28

Tertiary and primary windings

I_{3e} [A]	j_3 [A/mm^2]	j_1 [A/mm^2]	j_2 [A/mm^2]	j_R [A/mm^2]	W_{31} [J]	x_{k31} [Ω]	x_{k31} [%]	x_{k31} [Ω] [exp.]	Error [%]
2100	-2.045	0.16	0.252	0	1884	0.537	8.88	0.546	1.7
2100	-2.045	0.141	0.222	0.259	2186	0.623	10.3	0.618	-0.8
2100	-2.045	0.184	0.291	-0.338	1778	0.507	8.38	0.522	2.9

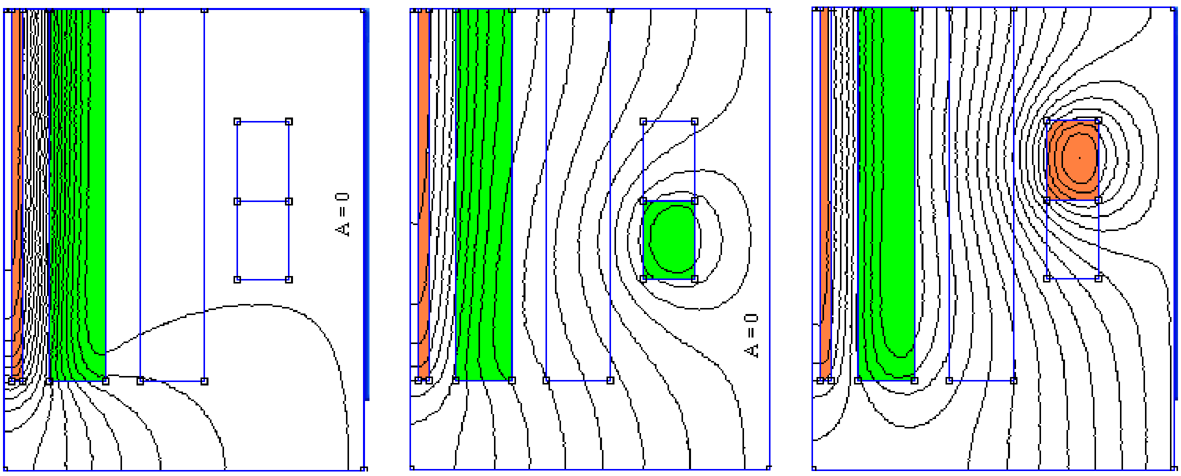


Principal tapping | Plus tapping | Minus tapping

Fig. 9. Magnetic field in inferior half window of the autotransformer for tertiary-secondary windings short-circuit test

Tertiary and secondary windings

I_{3e} [A]	j_3 [A/mm^2]	j_2 [A/mm^2]	j_R [A/mm^2]	W_{32} [J]	x_{k32} [Ω]	x_{k32} [%]	x_{k32} [Ω] [exp.]	Error [%]
2100	-2.045	0.436	0	1256	0.358	5.92	0.368	2.8
2100	-2.045	0.355	0.412	1648	0.47	7.76	0.472	0.4
2100	-2.045	0.567	-0.659	1925	0.55	9.07	0.506	-7.8



Principal tapping | Plus tapping | Minus tapping

Fig. 10. Magnetic field in inferior half window of the autotransformer for tertiary-secondary windings short-circuit test

Conclusions

1. In the case of regulating windings, the impedance voltage cannot more be accurately determined using classical methods and finite element method must be used.
2. At commercial frequency the transformer leakage magnetic flux can be well approximated with the static (zero frequency) flux.
3. In the transformer window the parallel to the columns line of symmetry can be considered as magnetic flux line with zero vector magnetic potential.
4. For screened tank transformers, in these approximations, the leakage magnetic flux can be considered axially symmetric and “axisymmetric solution” of Femm can be successfully used.
5. The calculated values of the impedance voltage, of the studied 400 MVA autotransformer, agree with the experimental ones with a precision less than 3 %, with one exception of almost 8 %, for minus tapping of tertiary and secondary windings, when the magnetic field is stronger near the vertical symmetry axis of the autotransformer window.

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