NUMERICAL SOLUTION FOR INITIAL MAGNETIC FIELD DISTRIBUTION AROUND RECTANGULAR BUS BARS

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Abstract. The transient magnetic field depends essentially on the initial magnetic field on the bar surface. In the paper, a numerical solution using FEM for initial magnetic field on the surface of arbitrary cross-section rectangular bar is presented. Very good agreement with previous results of numerical conformal mapping for the initial magnetic field around the bar is obtained for a current step and numerical validations of simple formulas for ratio of fields on the two sides of the bar are given.

Keywords: Bus Bar, FEM 2D, Static Magnetic Field.

INTRODUCTION

The transient electromagnetic field problem in two infinite-high and long non-magnetic bars is completely analytically solved in [1], considering the magnetic field on external sides of the bars equal to zero. In [2] these results were extended for the case of finite but high-enough bars, maintaining the assumption of one-dimensional field in central part of the bar, constantly distributed on the height and considering the external magnetic field no more zero. The transient magnetic field in conductor is theoretically solved in function of the ratio η of the magnetic fields on the middle of two sides of the bar. An analytical approximate formula for ratio η_0 of magnetic fields on these sides at initial moment ($t = 0$) is proposed and it shows that the value of η changes very little from $t = 0$ to $t = \infty$, considered to be constant on the height of the bars. Numerical validation of the results of [2] with 2-D FEM are presented in [3] where a mixed analytical-numerical solution is also proposed, using the parameter η given by numerical calculation. The paper [4] provides exact solutions for the parameters η and η_{av} (of average of magnetic fields) at $t = 0$ with conformal mapping method (CMM) for infinitely thin bars and in [5] is made an analysis using Schwartz-Christoffel (SC) toolbox for numerical conformal mapping from MATLAB and simple approximate formulas are proposed, that extend the applicability to the finite thickness bars. Based on these formulas, the average initial magnetic field between the bars is determined.

In this paper a numerical solution for the initial magnetic field is obtained with finite element method (FEM) using FEMM program at current step injection to short-circuited high-enough bars. The results are compared with previous numerical conformal mapping evaluation [5] and the simple formulas from [5] are verified.

PREVIOUS SOLUTIONS FOR INITIAL MAGNETIC FIELD

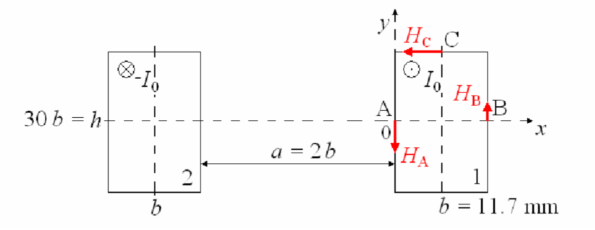

Figure 1. System of two parallels current carry bus bars

The initial magnetic field was analyzed for a system of two parallels rectangular bus bars, presented in Fig. 1 and it was quantified by the ratio η of the magnetic fields on the middle of two sides of the bar:

(1) $$\eta = \left| \frac{H_{\mathrm{yB}}}{H_{\mathrm{yA}}} \right| .$$

The transient magnetic field depends essentially on the initial magnetic field on the bar surface. At current step injection $I_0 = 2b_1$ (I_0 in Amperes if b_1 in meter) to the bus bars system, in the paper [4] are provided exact solutions for the parameters η and η_{av} (of average of magnetic fields) at $t = 0$ with conformal mapping method (CMM) for infinitely thin bars ($b / a \rightarrow 0$).

In [5] is made an analysis using Schwartz-Christoffel (SC) toolbox for numerical conformal mapping from MATLAB and simple approximate formulas are proposed using the Dwight parameter for determination of electrodynamic forces between rectangular bus bars in direct current (Eqn. 2). These formulas extend the applicability to the finite thickness bars ($b < h$).

The ratio η_0 of magnetic fields on the middle of two sides of the bar at initial moment ($t = 0$) is:

(2) $$\eta_0(x) = \left| \frac{H_{\mathrm{yB}}}{H_{\mathrm{yA}}} \right| = \frac{\frac{\pi}{2} - \mathrm{arctg}\left(\frac{x}{2}\right)}{\frac{\pi}{2} + \mathrm{arctg}\left(\frac{x}{2}\right)}, \qquad x = \frac{h+b}{a}, \qquad b < h .$$

In [5] is also provided an approximate formula for average value of η_0:

(3) $$\eta_{0\text{av}}\left(\frac{h}{a}\right)=\left|\frac{H_{\text{yBav}}}{H_{\text{yAav}}}\right|=\frac{\frac{\pi}{2}-\text{arctg}\left(\frac{h}{2a}\right)}{\frac{\pi}{2}+\text{arctg}\left(\frac{h}{2a}\right)}\cdot\left[0.86+0.18\frac{h}{a}-0.007\left(\frac{h}{a}\right)^2\right].$$

Similarly to the ratio η, is defined the ratio η_b of tangential components in points C and A (Fig. 1) and for its average is provided the formula:

(4) $$\eta_{b\text{av}}\left(\frac{b}{a}\right)=\left|\frac{H_{\text{xC}}}{H_{\text{yA}}}\right|=\frac{\pi-4\text{arctg}\left(\frac{b}{2a+b}\right)}{\pi+4\text{arctg}\left(\frac{h}{2a+3b}\right)}\cdot\left[1+\frac{h}{100a}+\frac{b}{a}\left(0.03+\frac{h}{2000a}\right)\right].$$

Using previous relations, the average initial magnetic field between the bus bars can be obtained [5]:

(5) $$H_{\text{yAav}}=\frac{I_0}{h(\eta_{\text{av}}+1)+2b\eta_{b\text{av}}}.$$

NUMERICAL SOLUTION FOR INITIAL MAGNETIC FIELD

FEMM model

The analysis of initial magnetic field was made with FEMM program in magnetostatic regime. It was analyzed a quarter of the model, choosing first order triangular elements, counting 140,367 nodes and 280,426 surface elements and an open boundary was simulated by using asymptotic conditions (Fig. 2). The expulsion of field lines from the conductor domain at the moment $t = 0$ was obtained by a trick consisting of approximate cancellation of its relative permeability ($\mu_r = 10^{-7}$).

The automation of calculation was obtained by running command files (scripts) realized using the parametric language LUA both for pre-processing / processing and post-processing, which did not eliminate interactive work.

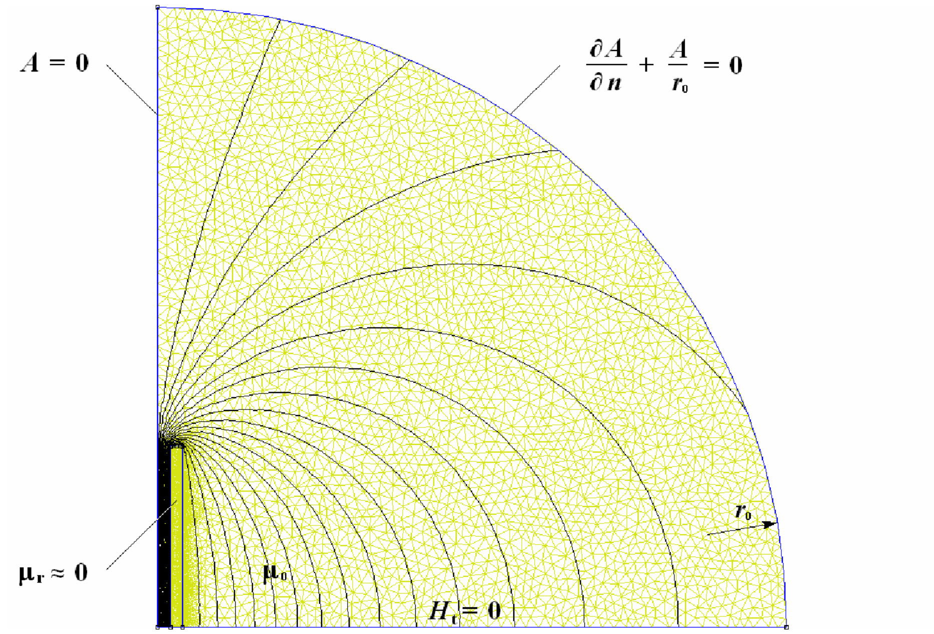

Figure 2. Studied domain and FEMM mesh (initial moment).

Numerical results for exterior initial magnetic field

The FEMM modeling for very thin bars (b / a = 0.01) validates quite well the results of work [4], where the ratio $\tilde{\eta}_0$ is calculated by the conformal mapping method (CMM), for configuration b / a = 0. In comparison, η_{0FEMM} = 0.03925 versus η_{0CMM} = = 0.03938 (error of 0.33 %) (Fig. 3).

Fig. 4 and Fig. 5 present a very accurate FEMM validation of the results for η_0 and η_{0av} obtained by Schwartz-Christoffel (SC) toolbox for numerical conformal mapping from MATLAB [5] for different $(h+b) / a$, respectively, h / a ratios (average errors less than 0.20 %). The plotting of analytical formulas (Eqn. 2) and (Eqn. 3) for b / a = 0 are added and compared to FEMM modeling for very thin bars (b / a = 0.01) (average error of 2.48 %, respectively, 1.87 %).

Another SC evaluation and FEMM validation can be seen in Fig. 6 and Fig. 7, remarking a very weak dependence of ratios η_b and η_{bav} on the ratio h / a (variation of 1.80 %, respectively, 11.19 % for h / a = 1 ÷ 10). SC results agree very well with numerical ones (average error less than 0.20 %). Also, it can bee seen that the applicability domain of the formula (Eqn. 4) can be considered $1 < b / a < 15$, where small errors compared with numerical results appear (average errors less than 1.00 %).

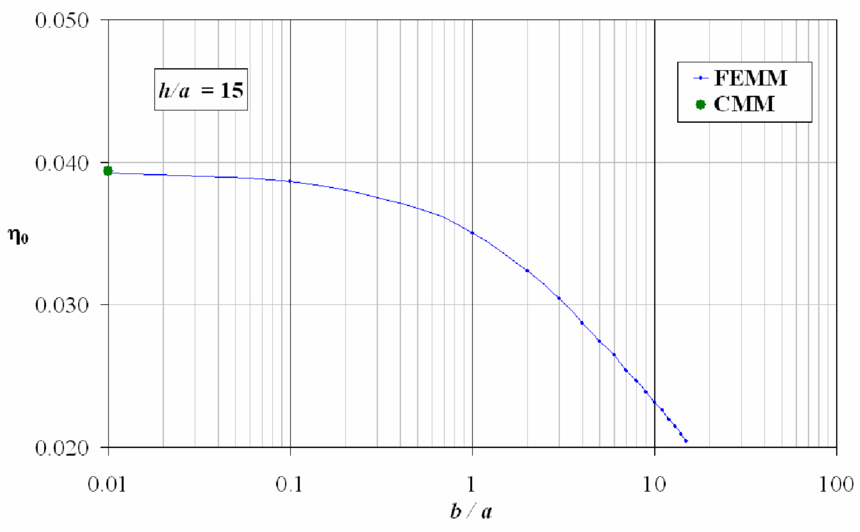

Figure 3. Ratio η_0 calculated by FEMM and CMM.

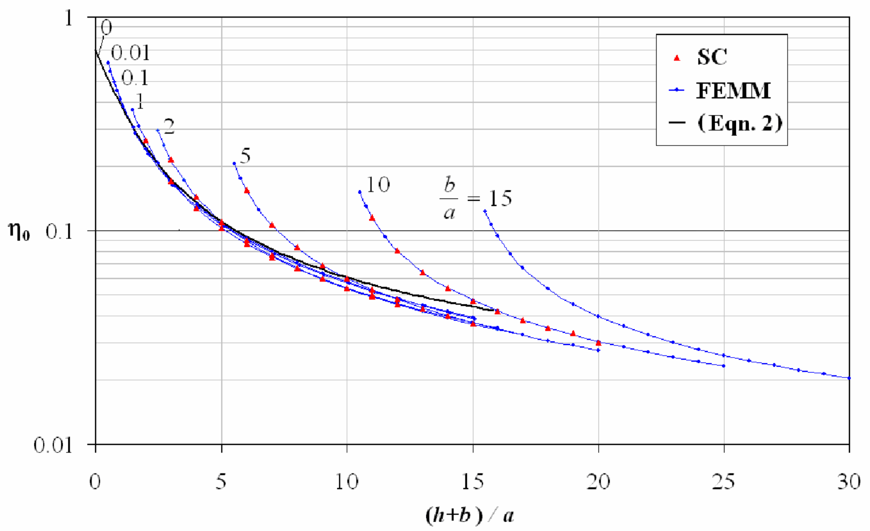

Figure 4. Bus bars geometry influence (*h/a*, *b/a*) on ratio η_0: numerical (SC, FEMM) and analytical (Eqn. 2) results.

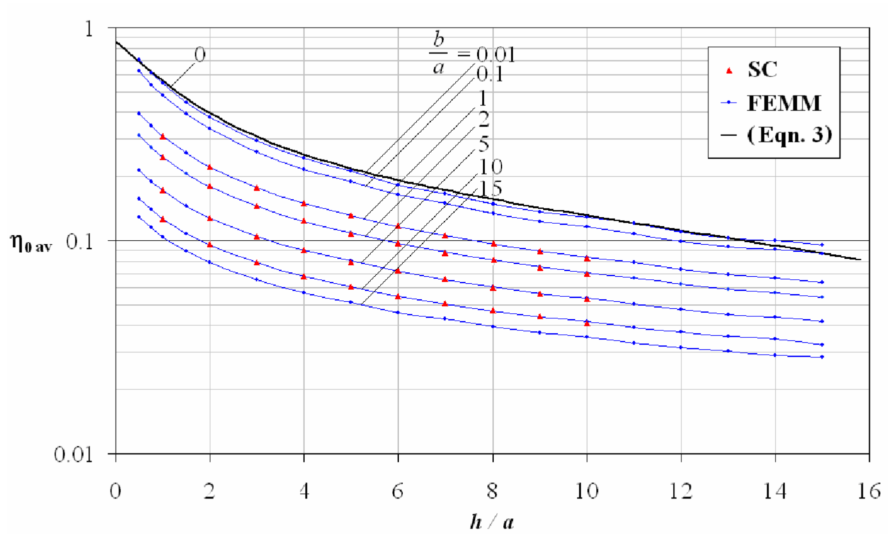

Figure 5. Bus bars geometry influence (*h/a*, *b/a*) on ratio η_{0av} : numerical (SC, FEMM) and analytical (Eqn. 3) results.

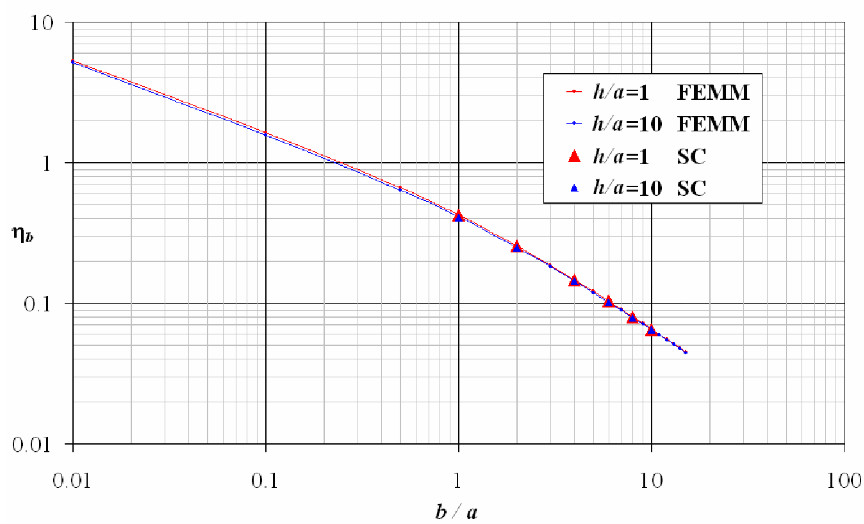

Figure 6. Bus bars geometry influence (*h/a*, *b/a*) on ratio η_b : numerical results (SC, FEMM).

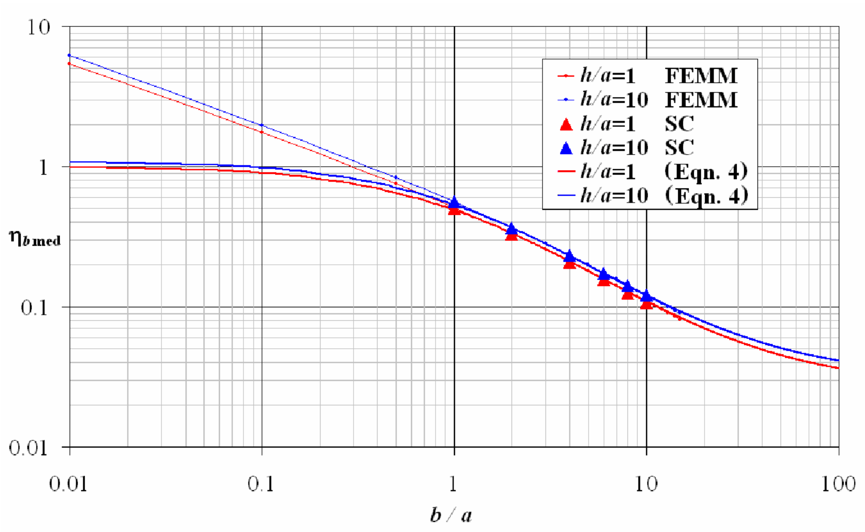

Figure 7. Bus bars geometry influence (*h/a*, *b/a*) on ratio η_{bav} : numerical (SC, FEMM) and analytical (Eqn. 4) results.

CONCLUSIONS

The numerical solution using finite element method (FEM) for initial magnetic field at current step injection to the bus bars system validates very well previous results, using conformal mapping method (CMM) [4], Schwartz-Christoffel (SC) toolbox for numerical conformal mapping from MATLAB and analytical formulas [5].

In terms of precision, SC and CMM are in very good agreement with FEM (relative errors less than 0.20 %, respectively, 0.33 %) followed by analytical formulas (relative errors about 2 %).

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